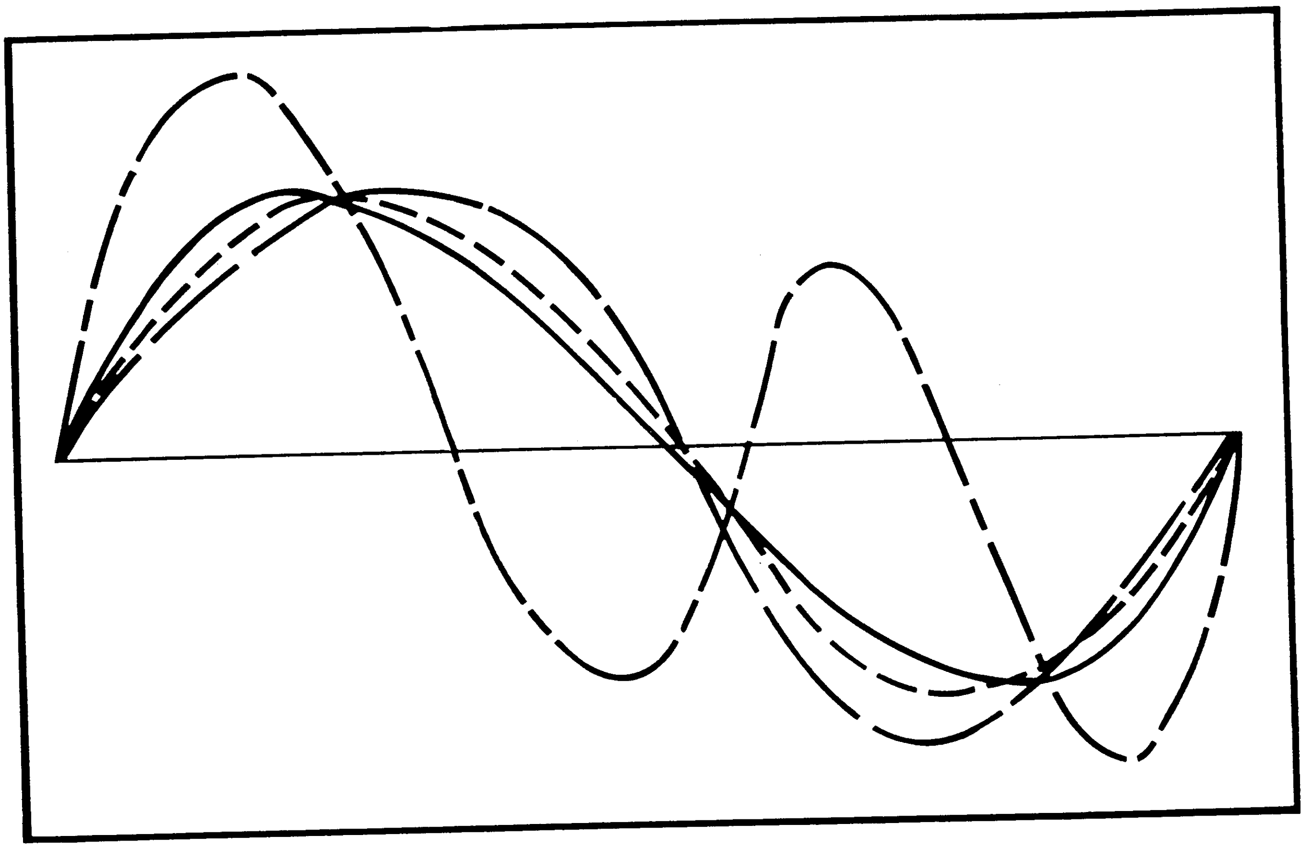
cam design



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EMERSON

a manual for engineers designers and draftsmen

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"Published by" Commercial Cam Division, Emerson Electric Company

PREFACE

This manual is intended for the engineer, designer or draftsman engaged in the design of moderate speed machinery in which cams are important elements. It presents an orderly design procedure for cams having fairly rigid follower systems. No attempt has been made to cover high speed systems in which deflection and vibration effect the efficiency of the operation.

In general it follows the practice of accepted authorities. Tabular data is used extensively to minimize tedious mathematical operations. Design examples are presented to illustrate the use of the tables and equations.

It contains certain novel features. The method of determining the radius of curvature, to the best of the writer's knowledge, has not been heretofore in the literature. Graphical methods for pressure angles and radii of curvature have been simplified to an orderly stepped procedure.

It is hoped that this manual will unravel some of the mysteries of cam design to the uninitiated.

Clyde H. Moon, P. E.
Dec. 20, 1961

Easton, Penna.

CONTENTS

Fundamental Mechanics

Introduction. Mechanics is a broad subject, but, fortunately, for cam design the band of topics is narrow. The subject matter in this section provides the necessary fundamentals. As the principles are basic, the references tabulated at the end of the section have been freely used and credit is hereby given.

Vector and Scalar Quantities. Vector quantities have both magnitude and direction. Examples are displacement, velocity, acceleration, and force. Scalar quantities have magnitude only. Examples are weight, mass, and volume. Vector quantities are shown in bold face type, thus: **A**, **B**, etc. Scalar magnitudes of vectors are shown in italics; thus, *A* is the scalar magnitude of vector **A**.

Addition of vectors creates a resultant vector. This may be done by the parallelogram method; Fig. A-1(a), or the triangle method, Fig. A-1(b), which show **R** as the resultant of the addition of vectors **A** and **B**. Three or more vectors may be added by the polygon method, Fig. A-1(c).

Vectors may be subtracted to give a resultant. To subtract vector **B** from vector **A**, the sign of **B** is reversed, which is the same as reversing its direction, and then added vectorially to **A**. See Fig. A-1(d).

A vector quantity may be resolved into components along any chosen axis as shown in Fig. A-1(e).

Scalar quantities are added and subtracted algebraically.

Displacement. Displacement is a vectorial quantity, occurring as translation or rotation or a combination of both. A moving body has translation when every line in the body remains parallel to its original position. It has rotation when all points travel in circles about an axis of rotation. Complex motions can be resolved into translation and rotation.

A translating body has linear displacement, velocity, and acceleration.

A rotating body has angular displacement, velocity, and acceleration.

Velocity. Velocity is the rate of change in displacement. When velocity is constant, equal increments of displacement occur in equal increments of time, i.e.: $v = y/t$. A point has variable velocity when displacements vary in magnitude, direction, or both, in equal time increments. The instantaneous velocity of such a point is determined by the derivative of displacement with respect to time: $v = dy/dt$. A point may have several different concurrent velocities which may be combined vectorially into a resultant. A resultant velocity can be resolved into components along chosen axes.

Acceleration. Acceleration is the rate of change in velocity. For a point moving in a straight line with constant acceleration, the relations between displacement, velocity, and acceleration are

$$y = \frac{v_F + v_o}{2} t \tag{1}$$

$$y = v_o t + \frac{at^2}{2} \tag{2}$$

$$v_F = v_o + at \tag{3}$$

$$v_F^2 = v_o^2 + 2ay \tag{4}$$

$$a = \frac{v_F - v_o}{t} \tag{5}$$

where t = interval of time; y = displacement; v_o = velocity at start of interval; v_F = velocity at end of interval; a = acceleration.

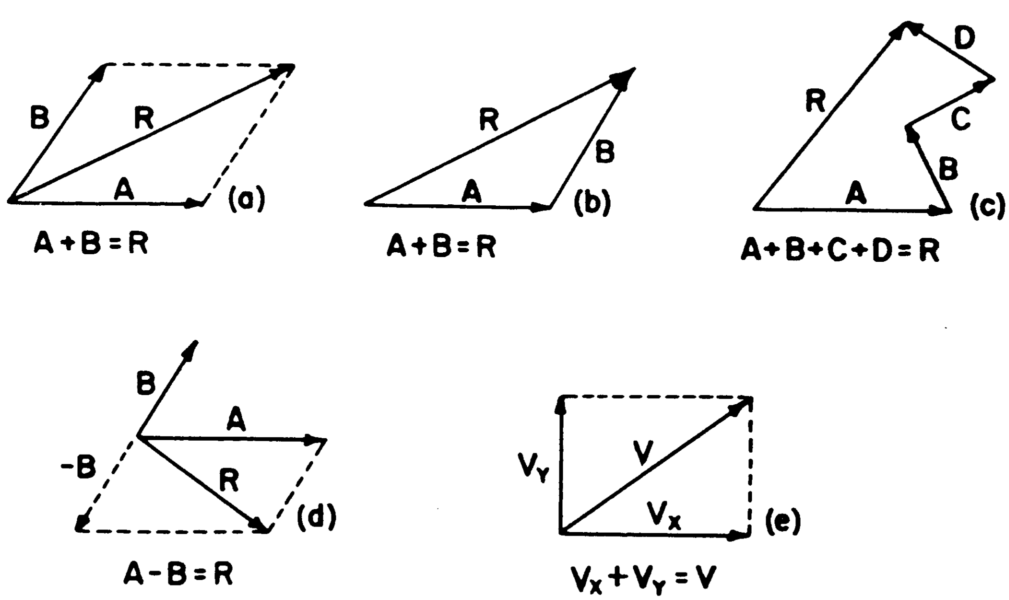


FIG. A-1

When acceleration varies in magnitude, direction or, both, the instantaneous acceleration is the derivative of velocity with respect to time: $a = dv/dt = d^2y/dt^2$.

A point may also have several different, concurrent, accelerations which may be combined vectorially into a resultant acceleration; and a resultant acceleration may be resolved into components along chosen axes.

Pulse. Pulse (or jerk) is the rate of change in acceleration. It is the derivative of acceleration with respect of time; i.e.: $p = da/dt = d^3y/dt^3$.

Rotation. A radian is an angle which subtends an arc equal in length to the radius of the arc.

Thus 2π *radians* = 360°; 1 *radian* = 57.3°.

In Fig. A-2(a), consider a point moving at a constant speed (v) in a circular path. As the point is continuously changing direction, the velocity is not constant. The change in velocity is indicated by the vectorial subtraction of V_1 from V_2. See Fig. A-2(b).

Acceleration, designated as normal acceleration, is directed toward the center with a magnitude $a = v^2/r$.

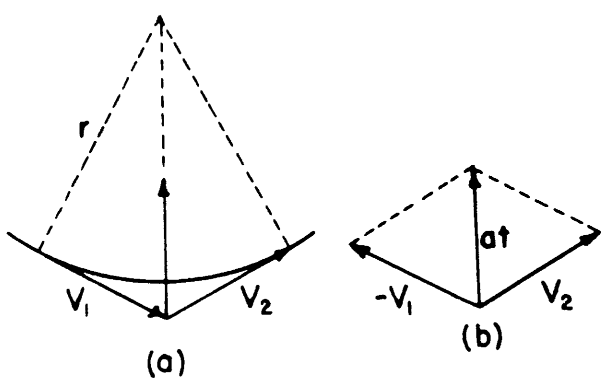


FIG. A-2

For constant angular acceleration, the relationship of angular displacement, velocity, and acceleration are as follows:

$$\phi = \frac{\omega_F + \omega_0}{2} t \qquad (6)$$

$$\phi = \omega_o t + \frac{\alpha t^2}{2} \qquad (7)$$

$$\omega_F = \omega_o + \alpha t \qquad (8)$$

$$\omega_F{}^2 = \omega_o{}^2 + 2\alpha\phi \qquad (9)$$

$$\alpha = \frac{\omega_F - \omega_0}{t} \qquad (10)$$

where t = interval of time; ϕ = angular displacement in radians; ω_o = angular velocity at start of time interval in radians per time unit; ω_F = angular velocity at end of time interval; α = angular acceleration in radians per unit time squared.

The instantaneous values of variable angular velocity and acceleration are determined by the derivatives with respect to time: $\omega = d\phi/dt$ and $\alpha = d\omega/dt$.

The relationships between linear and angular expressions are

$$s = r\phi \qquad (11)$$

$$v = r\omega \qquad (12)$$

$$a_n = \frac{v^2}{r} = r\omega^2 = v\omega \qquad (13)$$

$$a_t = r\alpha \qquad (14)$$

where s = arc length of circular path; a_n = normal component of acceleration; a_t = tangential component of acceleration; r = radius of circle; ϕ, ω, α, v as noted before.

Force and Mass. The concept of force and mass is provided by Newton's laws of motion.

1. *A body maintains its state of rest or uniform motion unless compelled by some force to change that state.*
2. *An unbalanced force acting on a body accelerates the body in the direction of the force. The acceleration produced is directly proportional to the force and inversely proportional to the mass of the body.*
3. *To every action or force there is an equal and opposite reaction.*

According to the second law, if an unbalanced force F imparts an acceleration a to a body, a different force F_1 will impart a different, but proportional, acceleration a_1. That is:

$$\frac{F}{F_1} = \frac{ma}{ma_1} \qquad (15)$$

where m is the mass of the body.

If a body falls freely, the unbalanced force is the weight (W) and the acceleration is g (386 in./sec.²). Substituting W for F_1 and g for a_1

$$\frac{F}{W} = \frac{a}{g}$$

and

$$F = \frac{W}{g}a = ma \qquad (16)$$

Force is a vector quantity; so two or more forces can be added vectorially to produce a resultant, which is the single force that will give the resultant acceleration.

When the resultant force is zero, the body is in equilibrium and no change occurs in the state of rest or motion of the body.

References

1. Theory and Problems of Engineering Mechanics. McLean and Nelson. Schaum Publishing Co., N. Y.
2. Physical Mechanics. G. H. Logan. Machine Design. April, 1956.

Cam Systems

Basic Elements. A cam system consists of four elements: cam, follower, follower system, and drive.

A cam is a mechanical part which imparts a prescribed motion to another part by direct contact. It may remain stationary, translate, or rotate.

The follower is the element directly contacting the cam. It may be of various shapes.

The follower system includes all the elements to which motion is imparted by the cam. They may be connected directly to the follower, or through linkage and gearing. The follower and follower system may translate or oscillate.

The drive consists of the prime mover, gears, cam shaft, etc. which impart motion to a rotating or translating cam, or to the follower system of a stationary cam.

Cam systems may be designed in a variety of physical forms. A number of the more commonly used forms, classified according to cam and follower types, are presented in this section. These are based largely on the more comprehensive catalog in the reference at the end of the section.

In this manual, only the open or disk cam, the closed or face-groove cam, and the cylindrical cam will be discussed, but the principles set forth are equally applicable to the other types.

Cam and Follower Classification

1. Sequence of Follower Operation. The three common types are shown diagrammatically in Fig. B-1.
   (a) Dwell-rise-dwell cam (D-R-D), Fig. B-1(a). This is the most common type. It has a dwell at the beginning and end of the rise.
   (b) Dwell-rise-return-dwell cam (D-R-R-D), Fig. B-1(b). There is no dwell between the rise and return.
   (c) Rise-return-rise cam (R-R-R), Fig. B-1(c). There are no dwells. This type has little application, as the motion is more adapted to an eccentric.

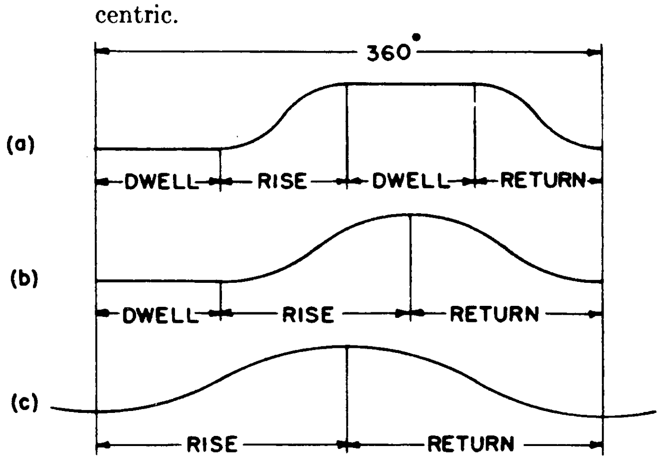


FIG. B-1

2. Follower Shape
   (a) Roller follower, Fig. B-2. This is the most commonly used follower. Pressure angles should be low to prevent jamming.
   (b) Knife-edge follower, Fig. B-3. This is of simple form but edge wears rapidly.
   (c) Flat-face follower, Fig. B-4. This type can be used with a steep cam, as it will not jam. Deflection or misalignment can cause high surface stress.
   (d) Spherical-face follower, Fig. B-5. Radiused face compensates for deflection or misalignment.

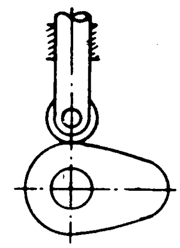


FIG. B-2

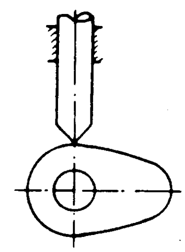


FIG. B-3

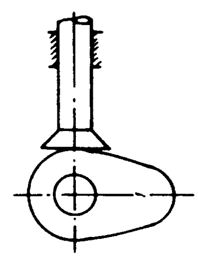


FIG. B-4

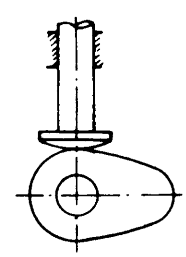


FIG. B-5

3. Follower Motion.
   (a) Translating follower, Fig. B-2. The follower moves in a straight line.
   (b) Swinging arm follower, Fig. B-6. The follower moves in a circular arc.
4. Follower Position.
   (a) On-center follower, Fig. B-2. Line of follower motion passes through axis of cam rotation.
   (b) Offset follower, Fig. B-7. Line of follower motion does not pass through axis of cam rotation. Offset should be in direction that reduces force components tending to cause jamming.

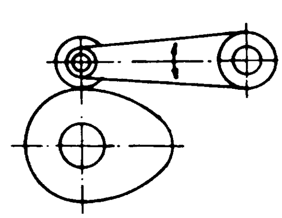


FIG. B-6

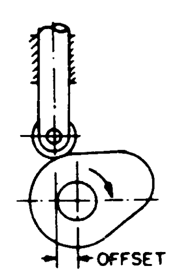


FIG. B-7

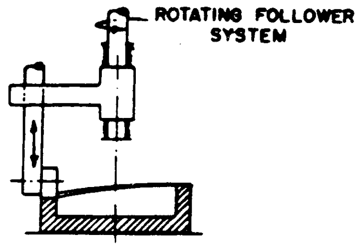


FIG. B-8

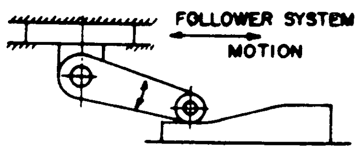


FIG. B-9

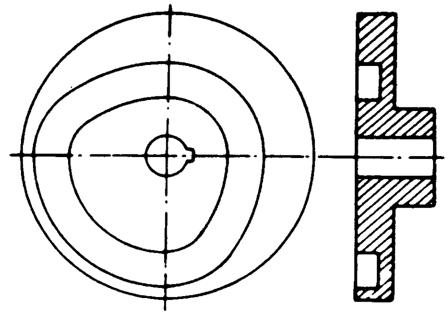


FIG. B-10

5. Cam Motion.
   (a) Rotating cam, Figs. B-2, 7, 10, 11. Rotation is usually of constant speed.
   (b) Translating cam, Figs. B-12, 13. The cam usually reciprocates in straight line motion.
   (c) Stationary cam, Figs. B-8, 9. The follower system rotates or translates.

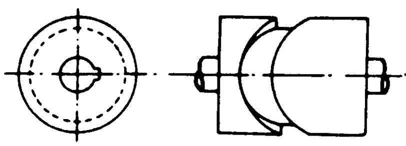


FIG. B-11

6. Cam Shape.
   (a) Open cam (also known as disk or plate cam), Figs. B-2, 6, 16 show examples. Translating or swinging arm follower must be constrained to maintain contact with cam profile.
   (b) Closed cam (also known as face-groove or positive cam), Fig. B-10. The groove constrains follower to positive action.
   (c) Cylindrical cam, Fig. B-11. The roller follower operates in a groove cut on the periphery of a cylinder. Follower may be translating or swinging arm type.

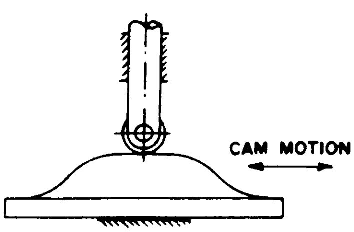


FIG. B-12

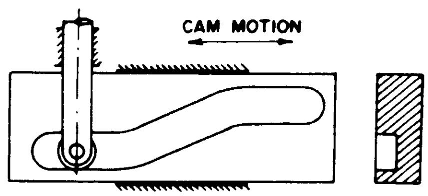


FIG. B-13

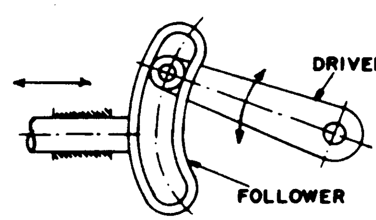


FIG. B-14

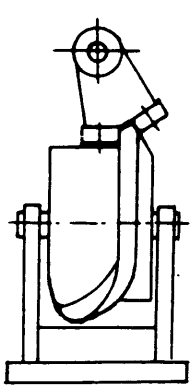


FIG. B-15

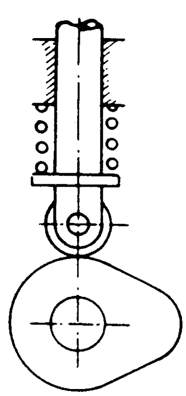


FIG. B-16

   (d) Wedge cam, Fig. B-12. Cam usually has reciprocating straight line motion.
   (e) Flat-plate cam, Fig. B-13. This is positive-drive version of the wedge cam.
   (f) Inverse cam, Fig. B-14. Normal functions of cam and follower are reversed. The most common example is the geneva motion.
   (g) Roller-gear drive cam, Fig. B-15. Projected ridge contacts dual roller followers. The Ferguson indexing cam is an example.
7. Follower Constraint.
   (a) Gravity constraint, Fig. B-2. Weight of follower system is sufficient to maintain contact.
   (b) Spring constraint, Fig. B-16. The spring must be strong enough to maintain contact.
   (c) Positive constraint, Figs. B-10, 11, 13. The groove maintains positive action.

Reference

1. Basic Cam Systems. H. A. Rothbart. Machine Design. May 31, 1956.

SECTION C

Cam Nomenclature

The Displacement Diagram is a rectangular coordinate layout of the follower motion in one cycle of cam operation. The rise of the cam is shown as the ordinate, the length of the abscissa being arbitrarily chosen. The abscissa is divided into equal cam angles or time divisions. A sketch of the displacement diagram is the first step in the development of the cam profile. See Fig. C-1(a).

The Velocity and Acceleration Diagrams are coordinate layouts of the magnitude of the velocities and accelerations. Typical diagrams are shown in Figs. C-1(b) and C-1(c).

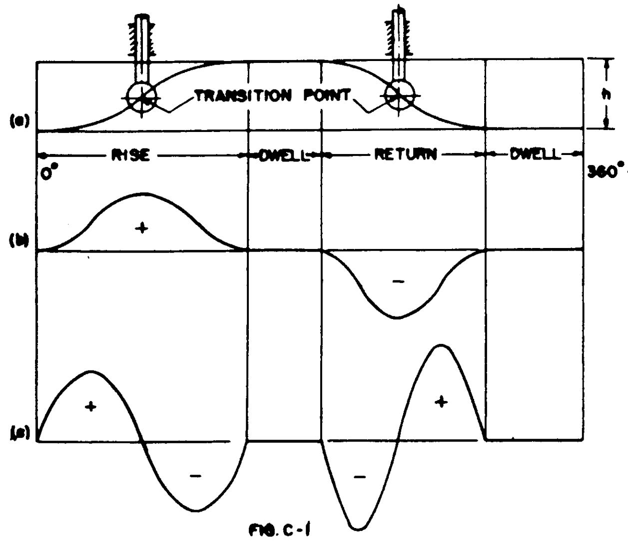


FIG. C-1
(a) Displacement; (b) Velocity; (c) Acceleration

The Transition Point is the point of maximum velocity.

The following definitions apply to Fig. C-2:

The Cam Profile is the working surface of cam in contact with the follower. In a closed or grooved cam there is an inner profile and an outer profile.

The Base Radius (R_b) is the smallest radius from the cam center to the cam profile.

The Trace Point is the center of the roller follower.

The Pitch Curve is the path of the trace point. This curve is usually determined first and the cam profile established by tangents to the roller follower.

The Minor Radius (R_o) is the smallest radius from the cam center to the pitch curve.

The Major Radius (R_N) is the largest radius from the cam center to the pitch curve.

The Pressure Angle (γ) is the angle at any point between the normal to the pitch curve and the instantaneous direction of the follower motion. This angle is important in cam design because it represents the steepness of the cam profile. Too steep a contour can cause jamming of the follower system in its guides, therefore, cams with radial followers are usually designed with a maximum pressure angle of approximately 30 degrees.

With a swinging arm follower the pressure angle is less important as it would be impossible to jam the system unless the line of action went through the pivot of the follower arm.

The Pitch Point is the point of maximum pressure angle. This is the start of design for minimum cam size. On cylindrical cams it is coincident with the transition point. On disk cams, because of the distortion resulting from converting the displacement diagram to radial divisions, it does not coincide with the transition point. However, for practical purposes these points on most cams can be assumed at the same point.

The Pitch Radius (R_p) has its center at the cam axis and passes through the pitch point. This radius is used for calculating a cam of minimum size for a given pressure angle.

The Radius of Curvature (R_c) at any point of the pitch curve is the radius of a circle, tangent to the curve, whose curvature is the same as that of the pitch curve at that point.

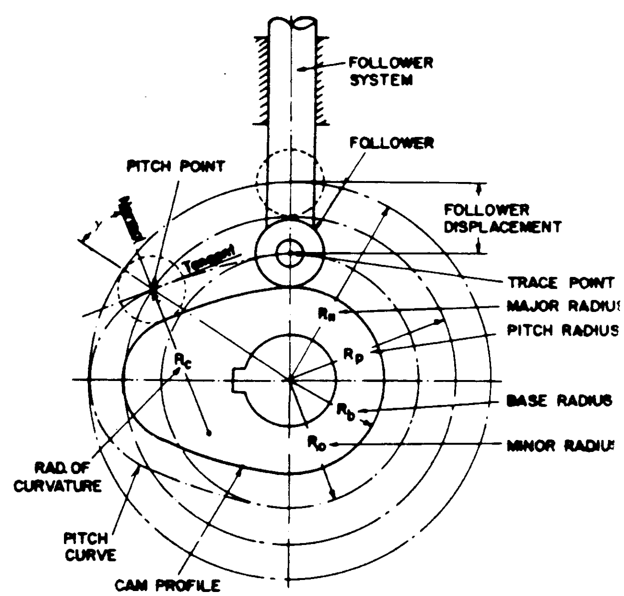


FIG. C-2

Reference

1. Cams--Design, Dynamics and Accuracy. H. A. Rothbart. John Wiley & Sons, Inc. 1956.

Basic Curves

Nomenclature

a = linear acceleration of follower (in./sec.^{2})
v = linear velocity of follower (in./sec.)
h = total displacement of follower (in.)
y = displacement of follower at any point (in.)
β = angular displacement of cam for displacement h (radians)
θ = angular displacement of cam for displacement y (radians)
t = time for cam to rotate through θ (sec.)
T = time for cam to rotate through β (sec.)

Classification. Basic curves are primarily of two classes: simple polynomial and trigonometric. The simple polynomial curves include the constant velocity or straight line, the constant acceleration or parabolic, and the cubic curves. Only the constant velocity and constant acceleration curves will be discussed. The trigonometric curves include the harmonic, the cycloidal, the double harmonic, and the elliptical. Only the harmonic and cycloidal will be discussed.

Constant Velocity. See Fig. D-1. This curve has a straight line displacement diagram. It has uniform displacement, constant velocity, and zero acceleration. At the terminals there is the impracticable condition of instantaneous change in velocity, resulting in theoretically infinite accelerations. This condition makes this curve undesirable except in combination with other curves. Characteristics of the constant velocity curve are:

$$\text{Displacement:} \quad y = h\frac{\theta}{\beta} \tag{1}$$

$$\text{Velocity:} \quad v = \frac{y}{t} = \frac{h}{T} \tag{2}$$

$$\text{Acceleration:} \quad a = 0 \tag{3}$$

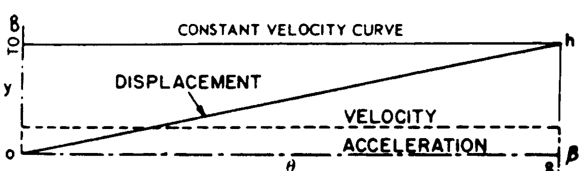


FIG. D-1

Constant Acceleration. See Fig. D-2. This curve, also known as parabolic or gravity curve, has constant positive and negative accelerations. It has an abrupt change of acceleration at the terminals and the transition point, which makes it undesirable except at low speeds. It provides the lowest acceleration of all curves for a given motion. In combination with other curves it can be used to advantage. For constant acceleration, the following equations are valid:

$$v = v_o + at \tag{4}$$

$$v^2 = v_o^2 + 2ay \tag{5}$$

$$y = v_o t + 0.5at^2 \tag{6}$$

$$y = 0.5(v_F + v_o)t \tag{7}$$

where v_o = initial velocity; v_F = final velocity; a = acceleration; y = displacement.

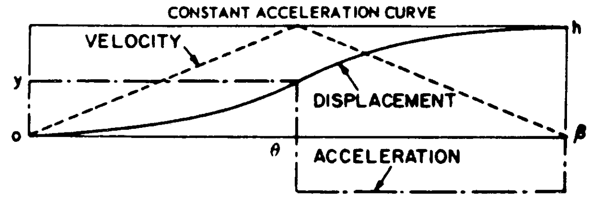


FIG. D-2

In terms of linear and angular displacements, its characteristics are as follows:

Cam angle θ from zero to 0.5β:

$$\text{Displacement:} \quad y = 2h\left(\frac{\theta}{\beta}\right)^2 \tag{8}$$

$$\text{Velocity:} \quad v = \frac{4h(\theta/\beta)}{T} \tag{9}$$

$$\text{Acceleration:} \quad a = \frac{4h}{T^2} \tag{10}$$

Cam angle θ from 0.5β to β:

$$\text{Displacement:} \quad y = h - 2h\left(1 - \frac{\theta}{\beta}\right)^2 \tag{11}$$

$$\text{Velocity:} \quad v = 4h\frac{[1 - (\theta/\beta)]}{T} \tag{12}$$

$$\text{Acceleration:} \quad a = -\frac{4h}{T^2} \tag{13}$$

Construction of the displacement diagram is shown in Fig. D-3.

Line AB represents the number of degrees through which the cam rotates to produce the desired displacement of the cam follower. It is divided into the desired number (N) of equal parts. Line AC represents to scale the displacement of the cam follower. Any line AD is drawn and divided into N parts of units 1, 3, 5, 7 —7, 5, 3, 1. Lines parallel to CD are drawn intersecting line AC. The intersection of horizontally projected points on AC with vertically projected points on AB locates the necessary points for the displacement curve.

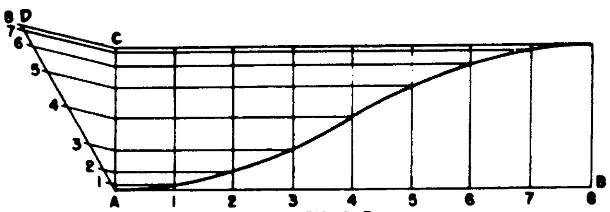


FIG. D-3
CONSTRUCTION DIAGRAM-CONSTANT ACCELERATION

Harmonic. See Fig. D-4. This curve is a definite improvement over the previous curves. It has a smooth continuous acceleration but has a sudden change at the dwell ends when used in a dwell-rise-dwell cam. This is objectionable at high speeds. In combination with other curves it is valuable for use in a dwell-rise-return-dwell cam.

The characteristics of this curve are:

$$\text{Displacement:} \quad y = 0.5h\left(1 - \cos \pi \frac{\theta}{\beta}\right) \tag{14}$$

$$\text{Velocity:} \quad v = \frac{0.5\pi h[\sin \pi(\theta/\beta)]}{T} \tag{15}$$

$$\text{Acceleration:} \quad a = \frac{0.5\pi^2 h[\cos \pi\ (\theta/\beta)]}{T^2} \tag{16}$$

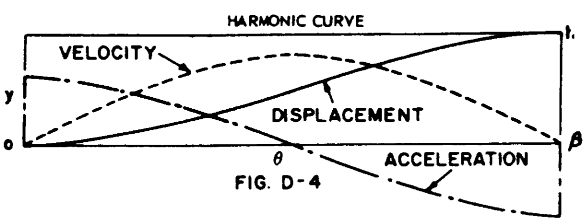


FIG. D-4

Construction of the displacement diagram is shown in Fig. D-5.

Line AB represents the number of degrees through which the cam rotates to produce the desired displacement of the follower. It is divided into the desired number (N) of equal parts. Line AC represents to scale the displacement of the cam follower. A semi-circle with radius equal to one-half the displacement is drawn as shown and divided into N equal parts. The intersection of horizontally projected points of the semi-circle with vertically projected points on AB locates the necessary points for the displacement curve.

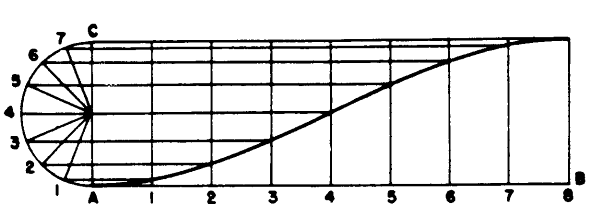


FIG. D-5
CONSTRUCTION DIAGRAM-HARMONIC

Cycloidal. See Fig. D-6. The cycloidal curve is developed from the path of a point on a circle which is rolled on a straight line. It is the most popular curve for high speeds, as there is no sudden change in acceleration at the dwell ends. It has the lowest vibration, wear, and stress of all the basic curves.

The characteristics of this curve are:

$$\text{Displacement:} \quad y = h\left[\frac{\theta}{\beta} - \frac{\sin 2\pi(\theta/\beta)}{2\pi}\right] \tag{17}$$

$$\text{Velocity:} \quad v = h\left[\frac{1 - \cos 2\pi(\theta/\beta)}{T}\right] \tag{18}$$

$$\text{Acceleration:} \quad a = h\left[\frac{2\pi \sin 2\pi(\theta/\beta)}{T^2}\right] \tag{19}$$

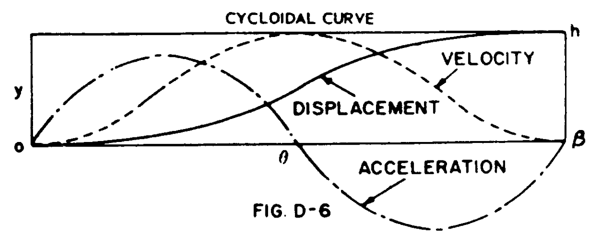


FIG. D-6

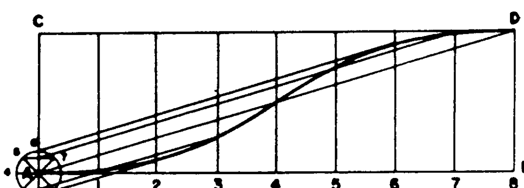


FIG. D-7
CONSTRUCTION DIAGRAM-CYCLOIDAL

Construction of the displacement diagram is shown in Fig. D-7.

Line AB represents the number of degrees through which the cam rotates to produce the desired displacement of the follower. It is divided into the desired number (N) of equal parts. Line AC represents to scale the displacement of the cam follower. With A as a center, a circle whose circumference is equal to AC is drawn, and divided into N equal parts. From the projection of these points on AC, lines are drawn parallel to AD, intersecting the vertical projections of the divisions of AB. These intersections locate the necessary points for the displacement curve.

Reference

1. Cams—Design, Dynamics and Accuracy, H. A. Rothbart. John Wiley & Sons, Inc. 1956.

Displacement, Velocity and Acceleration Tables for Basic Curves

Nomenclature

a = linear acceleration (in./sec.^{2})
v = linear velocity (in./sec.)
h = total displacement of follower (in.)
β = angular displacement of cam for displacement h (deg.)
y = displacement at any point (in.)
θ = angular displacement of cam for displacement y (deg.)
K = displacement factor at any point
C_v = velocity coefficient at any point
C_a = acceleration coefficient at any point
N = RPM of cam shaft

Basic Curve Factors. To simplify determination of the cam profile and the velocity and acceleration of follower, point-by-point factors have been calculated from the equations for the characteristics of each basic curve. These factors are contained in Tables E-1 through E-4. In addition, Tables E-5 and E-6 contain factors for the modified trapezoid and modified sine curves, which have not yet been discussed. These combination curves will be presented in Section N.

The tabulated values are dimensionless, the cam angle factor going from 0 to 120; the displacement factor from 0 to 1.

Equations. Using these factors, or coefficients, the characteristics of any basic curve can be stated in generalized form.

$$\text{Displacement: } y = Kh \tag{1}$$

$$\text{Velocity: } v = C_v h\left(\frac{6N}{\beta}\right) \tag{2}$$

$$\text{Acceleration: } a = C_a h\left(\frac{6N}{\beta}\right)^2 \tag{3}$$

The tables and equations are useful in many ways. They will be used for calculating profiles, pressure angles, radii of curvature, and inertial forces.

In Figs. E-1 through E-6 are shown characteristic diagrams of the basic curves in terms of the coefficients.

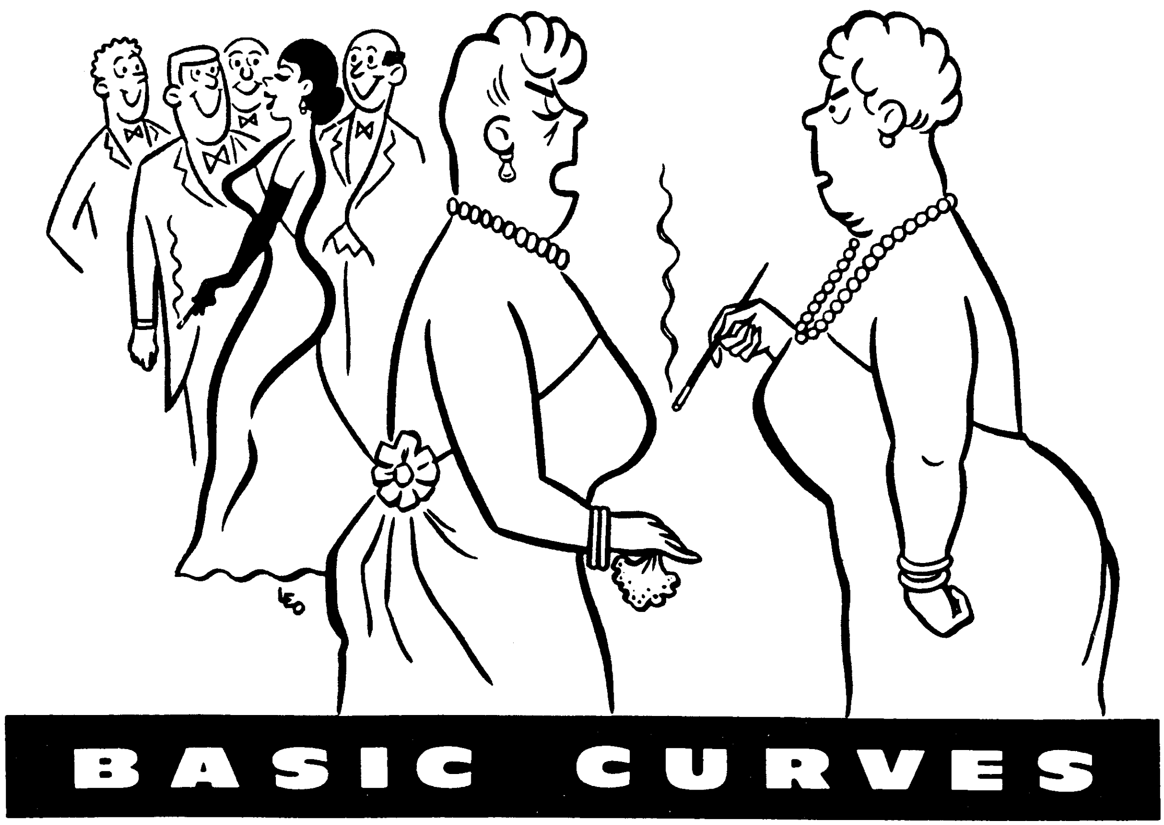


TABLE E-1. CONSTANT VELOCITY FACTORS

Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca
0	0.00000	1.0000	Zero	30	0.25000	1.0000	Zero	60	0.50000	1.0000	Zero	90	0.75000	1.0000	Zero
1	0.00833	"	"	31	0.25833	"	"	61	0.50833	"	"	91	0.75833	"	"
2	0.01667	"	"	32	0.26667	"	"	62	0.51667	"	"	92	0.76667	"	"
3	0.02500	"	"	33	0.27500	"	"	63	0.52500	"	"	93	0.77500	"	"
4	0.03333	"	"	34	0.28333	"	"	64	0.53333	"	"	94	0.78333	"	"
5	0.04167	"	"	35	0.29167	"	"	65	0.54167	"	"	95	0.79167	"	"
6	0.05000	"	"	36	0.30000	"	"	66	0.55000	"	"	96	0.80000	"	"
7	0.05833	"	"	37	0.30833	"	"	67	0.55833	"	"	97	0.80833	"	"
8	0.06667	"	"	38	0.31667	"	"	68	0.56667	"	"	98	0.81667	"	"
9	0.07500	"	"	39	0.32500	"	"	69	0.57500	"	"	99	0.82500	"	"
10	0.08333	"	"	40	0.33333	"	"	70	0.58333	"	"	100	0.83333	"	"
11	0.09167	"	"	41	0.34167	"	"	71	0.59167	"	"	101	0.84167	"	"
12	0.10000	"	"	42	0.35000	"	"	72	0.60000	"	"	102	0.85000	"	"
13	0.10833	"	"	43	0.35833	"	"	73	0.60833	"	"	103	0.85833	"	"
14	0.11667	"	"	44	0.36667	"	"	74	0.61667	"	"	104	0.86667	"	"
15	0.12500	"	"	45	0.37500	"	"	75	0.62500	"	"	105	0.87500	"	"
16	0.13333	"	"	46	0.38333	"	"	76	0.63333	"	"	106	0.88333	"	"
17	0.14167	"	"	47	0.39167	"	"	77	0.64167	"	"	107	0.89167	"	"
18	0.15000	"	"	48	0.40000	"	"	78	0.65000	"	"	108	0.90000	"	"
19	0.15833	"	"	49	0.40833	"	"	79	0.65833	"	"	109	0.90833	"	"
20	0.16667	"	"	50	0.41667	"	"	80	0.66667	"	"	110	0.91667	"	"
21	0.17500	"	"	51	0.42500	"	"	81	0.67500	"	"	111	0.92500	"	"
22	0.18333	"	"	52	0.43333	"	"	82	0.68333	"	"	112	0.93333	"	"
23	0.19167	"	"	53	0.44167	"	"	83	0.69167	"	"	113	0.94167	"	"
24	0.20000	"	"	54	0.45000	"	"	84	0.70000	"	"	114	0.95000	"	"
25	0.20833	"	"	55	0.45833	"	"	85	0.70833	"	"	115	0.95833	"	"
26	0.21667	"	"	56	0.46667	"	"	86	0.71667	"	"	116	0.96667	"	"
27	0.22500	"	"	57	0.47500	"	"	87	0.72500	"	"	117	0.97500	"	"
28	0.23333	"	"	58	0.48333	"	"	88	0.73333	"	"	118	0.98333	"	"
29	0.24167	"	"	59	0.49167	"	"	89	0.74167	"	"	119	0.99167	"	"
30	0.25000	"	"	60	0.50000	"	"	90	0.75000	"	"	120	1.00000	"	"

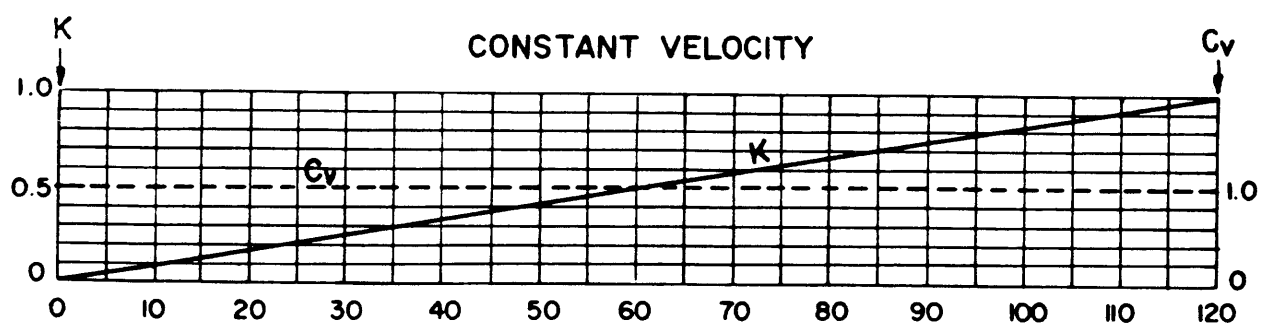


FIG. E-1

TABLE E-2. CONSTANT ACCELERATION FACTORS

Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca
0	0.00000	0.0000	4.000	30	0.12500	1.0000	4.000	60	0.50000	2.0000	-4.000	90	0.87500	1.0000	-4.000
1	0.00014	0.0333	"	31	0.13347	1.0333	"	61	0.51653	1.9667	"	91	0.88319	0.9667	"
2	0.00056	0.0667	"	32	0.14222	1.0667	"	62	0.53278	1.9333	"	92	0.89111	0.9333	"
3	0.00125	0.1000	"	33	0.15125	1.1000	"	63	0.54875	1.9000	"	93	0.89875	0.9000	"
4	0.00225	0.1333	"	34	0.16056	1.1333	"	64	0.56444	1.8667	"	94	0.90611	0.8667	"
5	0.00347	0.1667	"	35	0.17014	1.1667	"	65	0.57986	1.8333	"	95	0.91319	0.8333	"
6	0.00500	0.2000	"	36	0.13000	1.2000	"	66	0.59500	1.8000	"	96	0.92000	0.8000	"
7	0.00681	0.2333	"	37	0.19014	1.2333	"	67	0.60986	1.7667	"	97	0.92653	0.7667	"
8	0.00889	0.2667	"	38	0.20056	1.2667	"	68	0.62444	1.7333	"	98	0.93278	0.7333	"
9	0.01125	0.3000	"	39	0.21125	1.3000	"	69	0.63875	1.7000	"	99	0.93875	0.7000	"
10	0.01389	0.3333	"	40	0.22222	1.3333	"	70	0.65278	1.6667	"	100	0.94444	0.6667	"
11	0.01681	0.3667	"	41	0.23347	1.3667	"	71	0.66653	1.6333	"	101	0.94986	0.6333	"
12	0.02000	0.4000	"	42	0.24500	1.4000	"	72	0.68000	1.6000	"	102	0.95500	0.6000	"
13	0.02347	0.4333	"	43	0.25681	1.4333	"	73	0.69319	1.5667	"	103	0.95986	0.5667	"
14	0.02722	0.4667	"	44	0.26889	1.4667	"	74	0.70611	1.5333	"	104	0.96444	0.5333	"
15	0.03125	0.5000	"	45	0.28125	1.5000	"	75	0.71875	1.5000	"	105	0.96875	0.5000	"
16	0.03556	0.5333	"	46	0.29389	1.5333	"	76	0.73111	1.4667	"	106	0.97278	0.4667	"
17	0.04014	0.5667	"	47	0.30681	1.5667	"	77	0.74319	1.4333	"	107	0.97653	0.4333	"
18	0.04500	0.6000	"	48	0.32000	1.6000	"	78	0.75500	1.4000	"	108	0.98000	0.4000	"
19	0.05014	0.6333	"	49	0.33347	1.6333	"	79	0.76653	1.3667	"	109	0.98319	0.3667	"
20	0.05556	0.6667	"	50	0.34722	1.6667	"	80	0.77778	1.3333	"	110	0.98611	0.3333	"
21	0.06125	0.7000	"	51	0.36125	1.7000	"	81	0.78875	1.3000	"	111	0.98875	0.3000	"
22	0.06722	0.7333	"	52	0.37556	1.7333	"	82	0.79944	1.2667	"	112	0.99111	0.2667	"
23	0.07347	0.7667	"	53	0.39014	1.7667	"	83	0.80986	1.2333	"	113	0.99319	0.2333	"
24	0.08000	0.8000	"	54	0.40500	1.8000	"	84	0.82000	1.2000	"	114	0.99500	0.2000	"
25	0.08681	0.8333	"	55	0.42014	1.8333	"	85	0.82986	1.1667	"	115	0.99653	0.1667	"
26	0.09389	0.8667	"	56	0.43556	1.8667	"	86	0.83944	1.1333	"	116	0.99775	0.1333	"
27	0.10125	0.9000	"	57	0.45125	1.9000	"	87	0.84875	1.1000	"	117	0.99875	0.1000	"
28	0.10889	0.9333	"	58	0.46722	1.9333	"	88	0.85778	1.0667	"	118	0.99944	0.0667	"
29	0.11681	0.9667	"	59	0.48347	1.9667	"	89	0.86653	1.0333	"	119	0.99986	0.0333	"
30	0.12500	1.0000	"	60	0.50000	2.0000	"	90	0.87500	1.0000	"	120	1.00000	0.0000	"

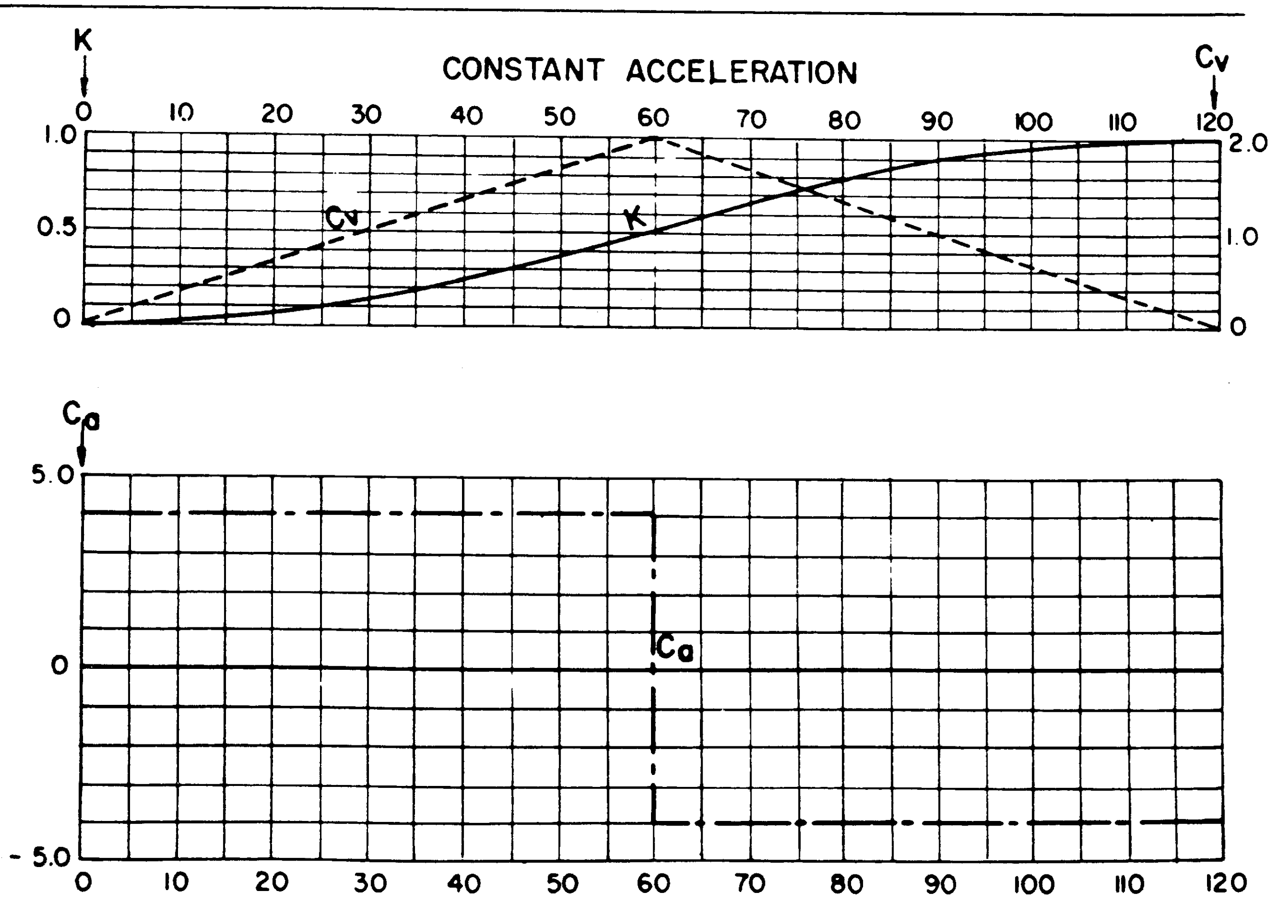


FIG. E-2

TABLE E-3. HARMONIC FACTORS

Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca
0	0.00000	0.0000	4.9348	30	0.14644	1.1107	3.4895	60	0.50000	1.5708	0.0000	90	0.85356	1.1107	-3.4895
1	0.00017	0.0411	4.9331	31	0.15582	1.1394	3.3969	61	0.51309	1.5703	-0.1292	91	0.86269	1.0813	-3.5796
2	0.00069	0.0822	4.9281	32	0.16543	1.1673	3.3020	62	0.52617	1.5686	-0.2582	92	0.87157	1.0511	-3.6673
3	0.00154	0.1232	4.9196	33	0.17527	1.1945	3.2049	63	0.53923	1.5660	-0.3872	93	0.88021	1.0202	-3.7525
4	0.00274	0.1642	4.9078	34	0.18543	1.2207	3.1056	64	0.55227	1.5622	-0.5158	94	0.88858	0.9885	-3.8351
5	0.00428	0.2050	4.8926	35	0.19562	1.2462	3.0041	65	0.56527	1.5574	-0.6441	95	0.89668	0.9562	-3.9150
6	0.00615	0.2457	4.8741	36	0.20611	1.2865	2.9006	66	0.57822	1.5515	-0.7719	96	0.90451	0.9233	-3.9924
7	0.00837	0.2862	4.8522	37	0.21679	1.2945	2.7951	67	0.59112	1.5469	-0.8993	97	0.91207	0.8897	-4.0669
8	0.01092	0.3266	4.8270	38	0.22768	1.3174	2.6877	68	0.60396	1.5365	-1.0260	98	0.91934	0.8555	-4.1387
9	0.01381	0.3667	4.7985	39	0.23875	1.3393	2.5784	69	0.61672	1.5274	-1.1520	99	0.92632	0.8207	-4.2076
10	0.01704	0.4066	4.7666	40	0.25000	1.3604	2.4674	70	0.62941	1.5173	-1.2772	100	0.93302	0.7854	-4.2737
11	0.02059	0.4461	4.7316	41	0.26142	1.3804	2.3547	71	0.64201	1.5061	-1.4015	101	0.93941	0.7495	-4.3368
12	0.02447	0.4854	4.6933	42	0.27300	1.3996	2.2404	72	0.65451	1.4939	-1.5250	102	0.94550	0.7131	-4.3970
13	0.02868	0.5243	4.6518	43	0.28474	1.4178	2.1245	73	0.66691	1.4807	-1.6473	103	0.95129	0.6762	-4.4541
14	0.03321	0.5629	4.6071	44	0.29663	1.4350	2.0072	74	0.67919	1.4665	-1.7685	104	0.95677	0.6389	-4.5082
15	0.03806	0.6011	4.5592	45	0.30866	1.4564	1.8884	75	0.69134	1.4564	-1.8884	105	0.96194	0.6011	-4.5592
16	0.04323	0.6389	4.5082	46	0.32081	1.4665	1.7685	76	0.70337	1.4350	-2.0072	106	0.96679	0.5629	-4.6071
17	0.04871	0.6762	4.4541	47	0.33309	1.4807	1.6473	77	0.71526	1.4178	-2.1245	107	0.97132	0.5243	-4.6518
18	0.05450	0.7131	4.3970	48	0.34549	1.4939	1.5250	78	0.72700	1.3996	-2.2404	108	0.97553	0.4854	-4.6933
19	0.06059	0.7495	4.3368	49	0.35799	1.5061	1.4015	79	0.73858	1.3804	-2.3547	109	0.97941	0.4461	-4.7316
20	0.06698	0.7854	4.2737	50	0.37059	1.5173	1.2772	80	0.75000	1.3604	-2.4674	110	0.98296	0.4066	-4.7666
21	0.07368	0.8207	4.2076	51	0.38328	1.5274	1.1520	81	0.76125	1.3393	-2.5784	111	0.98619	0.3667	-4.7985
22	0.08066	0.8555	4.1387	52	0.39604	1.5365	1.0260	82	0.77232	1.3174	-2.6877	112	0.98908	0.3266	-4.8270
23	0.08793	0.8897	4.0669	53	0.40888	1.5469	0.8993	83	0.78321	1.2945	-2.7951	113	0.99163	0.2862	-4.8522
24	0.09549	0.9233	3.9924	54	0.42178	1.5515	0.7719	84	0.79389	1.2865	-2.9006	114	0.99385	0.2457	-4.8741
25	0.10332	0.9562	3.9150	55	0.43478	1.5574	0.6441	85	0.80438	1.2462	-3.0041	115	0.99572	0.2050	-4.8926
26	0.11142	0.9885	3.8351	56	0.44773	1.5622	0.5158	86	0.81466	1.2207	-3.1056	116	0.99726	0.1642	-4.9078
27	0.11979	1.0202	3.7525	57	0.46077	1.5660	0.3872	87	0.82473	1.1945	-3.2049	117	0.99846	0.1232	-4.9196
28	0.12843	1.1511	3.6673	58	0.47383	1.5686	0.2583	88	0.83457	1.1673	-3.3020	118	0.99931	0.0822	-4.9281
29	0.13731	1.0813	3.5796	59	0.48691	1.5703	0.1292	89	0.84418	1.1394	-3.3969	119	0.99983	0.0411	-4.9331
30	0.14644	1.1107	3.4895	60	0.50000	1.5708	0.0000	90	0.85356	1.1107	-3.4895	120	1.00000	0.0000	-4.9348

$L = \pi/2$

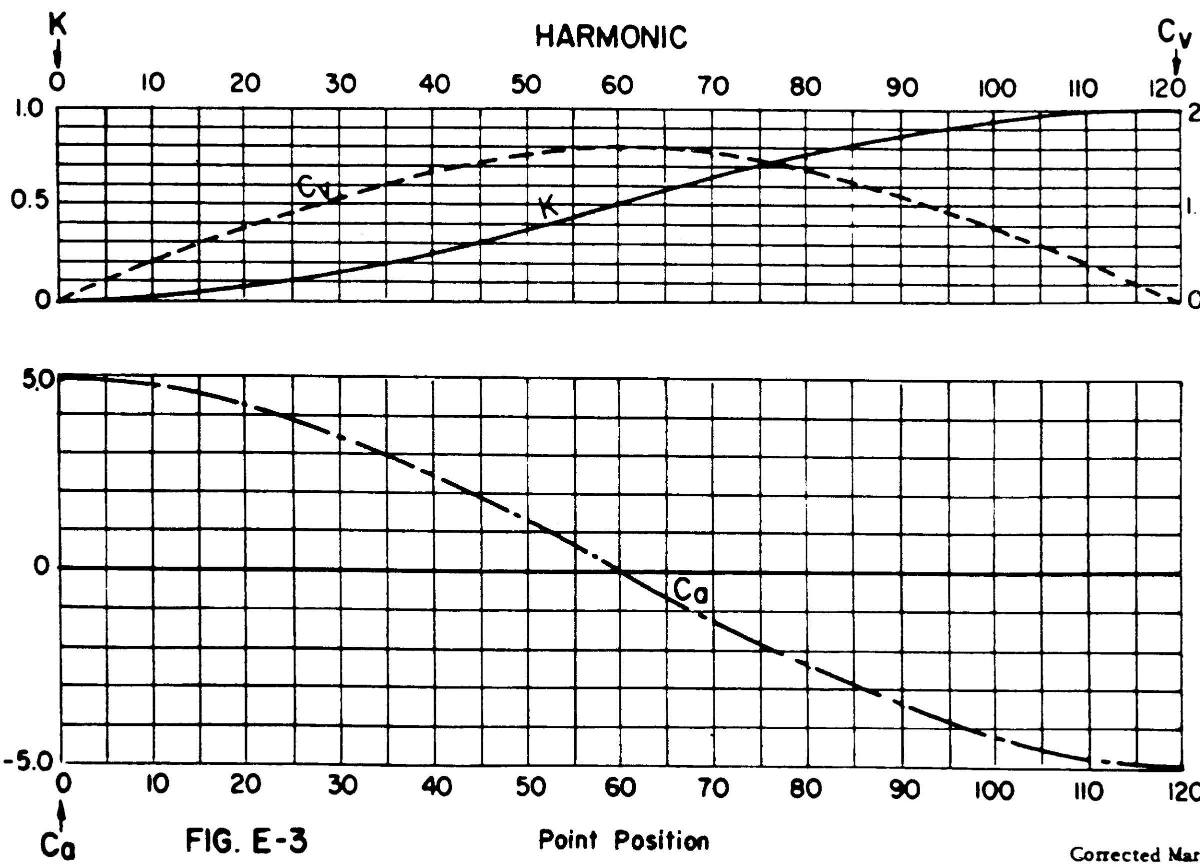


FIG. E-3

Corrected March 1962

TABLE E-4. CYCLOIDAL FACTORS

Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca
0	0.00000	0.0000	0.0000	30	0.09085	1.0000	6.2832	60	0.50000	2.0000	0.0000	90	0.90915	1.0000	-6.2832
1	0.00001	0.0014	0.3289	31	0.09940	1.0523	6.2746	61	0.51666	1.9986	-0.3289	91	0.91726	0.9477	-6.2746
2	0.00003	0.0055	0.6568	32	0.10839	1.1045	6.2488	62	0.53331	1.9945	-0.6568	92	0.92495	0.8955	-6.2488
3	0.00010	0.0123	0.9829	33	0.11781	1.1564	6.2059	63	0.54990	1.9877	-0.9829	93	0.93219	0.8436	-6.2059
4	0.00024	0.0218	1.3063	34	0.12766	1.2079	6.1459	64	0.56642	1.9782	-1.3063	94	0.93900	0.7921	-6.1459
5	0.00048	0.0341	1.6262	35	0.13794	1.2588	6.0691	65	0.58286	1.9659	-1.6262	95	0.94540	0.7412	-6.0691
6	0.00082	0.0489	1.9416	36	0.14864	1.3090	5.9757	66	0.59918	1.9511	-1.9416	96	0.95136	0.6910	-5.9757
7	0.00130	0.0664	2.2517	37	0.15975	1.3584	5.8659	67	0.61536	1.9336	-2.2517	97	0.95691	0.6416	-5.8659
8	0.00194	0.0865	2.5556	38	0.17128	1.4067	5.7400	68	0.63140	1.9135	-2.5556	98	0.96206	0.5933	-5.7400
9	0.00275	0.1090	2.8525	39	0.18320	1.4540	5.5984	69	0.64725	1.8910	-2.8525	99	0.96680	0.5460	-5.5984
10	0.00375	0.1340	3.1416	40	0.19550	1.5000	5.4414	70	0.66291	1.8660	-3.1416	100	0.97116	0.5000	-5.4414
11	0.00499	0.1613	3.4221	41	0.20820	1.5446	5.2695	71	0.67835	1.8387	-2.4221	101	0.97514	0.4554	-5.2695
12	0.00645	0.1910	3.6931	42	0.22124	1.5878	5.0832	72	0.69355	1.8090	-3.6931	102	0.97876	0.4122	-5.0832
13	0.00817	0.2228	3.9541	43	0.23465	1.6293	4.8830	73	0.70849	1.7772	-3.9541	103	0.98201	0.3707	-4.8830
14	0.01018	0.2569	4.2043	44	0.24840	1.6691	4.6693	74	0.72316	1.7431	-4.2043	104	0.98494	0.3309	-4.6693
15	0.01246	0.2929	4.4429	45	0.26246	1.7071	4.4429	75	0.73754	1.7071	-4.4429	105	0.98754	0.2929	-4.4429
16	0.01506	0.3309	4.6693	46	0.27684	1.7431	4.2043	76	0.75160	1.6691	-4.6693	106	0.98982	0.2569	-4.2043
17	0.01799	0.3707	4.8830	47	0.29151	1.7772	3.9541	77	0.76535	1.6293	-4.8830	107	0.99183	0.2228	-3.9541
18	0.02124	0.4122	5.0832	48	0.30645	1.8090	3.6931	78	0.77876	1.5878	-5.0832	108	0.99355	0.1910	-3.6931
19	0.02486	0.4554	5.2695	49	0.32165	1.8387	3.4221	79	0.79180	1.5446	-5.2695	109	0.99501	0.1613	-3.4221
20	0.02884	0.5000	5.4414	50	0.33709	1.8660	3.1416	80	0.80450	1.5000	-5.4414	110	0.99625	0.1340	-3.1416
21	0.03320	0.5460	5.5984	51	0.35275	1.8910	2.8525	81	0.81680	1.4540	-5.5984	111	0.99725	0.1090	-2.8525
22	0.03794	0.5933	5.7400	52	0.36860	1.9135	2.5556	82	0.82872	1.4067	-5.7400	112	0.99806	0.0865	-2.5556
23	0.04309	0.6416	5.8659	53	0.38464	1.9336	2.2517	83	0.84025	1.3584	-5.8659	113	0.99870	0.0664	-2.2517
24	0.04864	0.6910	5.9757	54	0.40082	1.9511	1.9416	84	0.85136	1.3090	-5.9757	114	0.99918	0.0489	-1.9416
25	0.05460	0.7412	6.0691	55	0.41714	1.9659	1.6262	85	0.86206	1.2588	-6.0691	115	0.99952	0.0341	-1.6262
26	0.06100	0.7921	6.1459	56	0.43358	1.9782	1.3063	86	0.87234	1.2079	-6.1459	116	0.99976	0.0218	-1.3063
27	0.06781	0.8436	6.2059	57	0.45010	1.9877	0.9829	87	0.88219	1.1564	-6.2059	117	0.99990	0.0123	-0.9829
28	0.07505	0.8955	6.2488	58	0.46669	1.9945	0.6568	88	0.89161	1.1045	-6.2488	118	0.99997	0.0055	-0.6568
29	0.08274	0.9477	6.2746	59	0.48334	1.9986	0.3289	89	0.90060	1.0523	-6.2746	119	0.99999	0.0014	-0.3289
30	0.09085	1.0000	6.2832	60	0.50000	2.0000	0.0000	90	0.90915	1.0000	-6.2832	120	1.00000	0.0000	0.0000

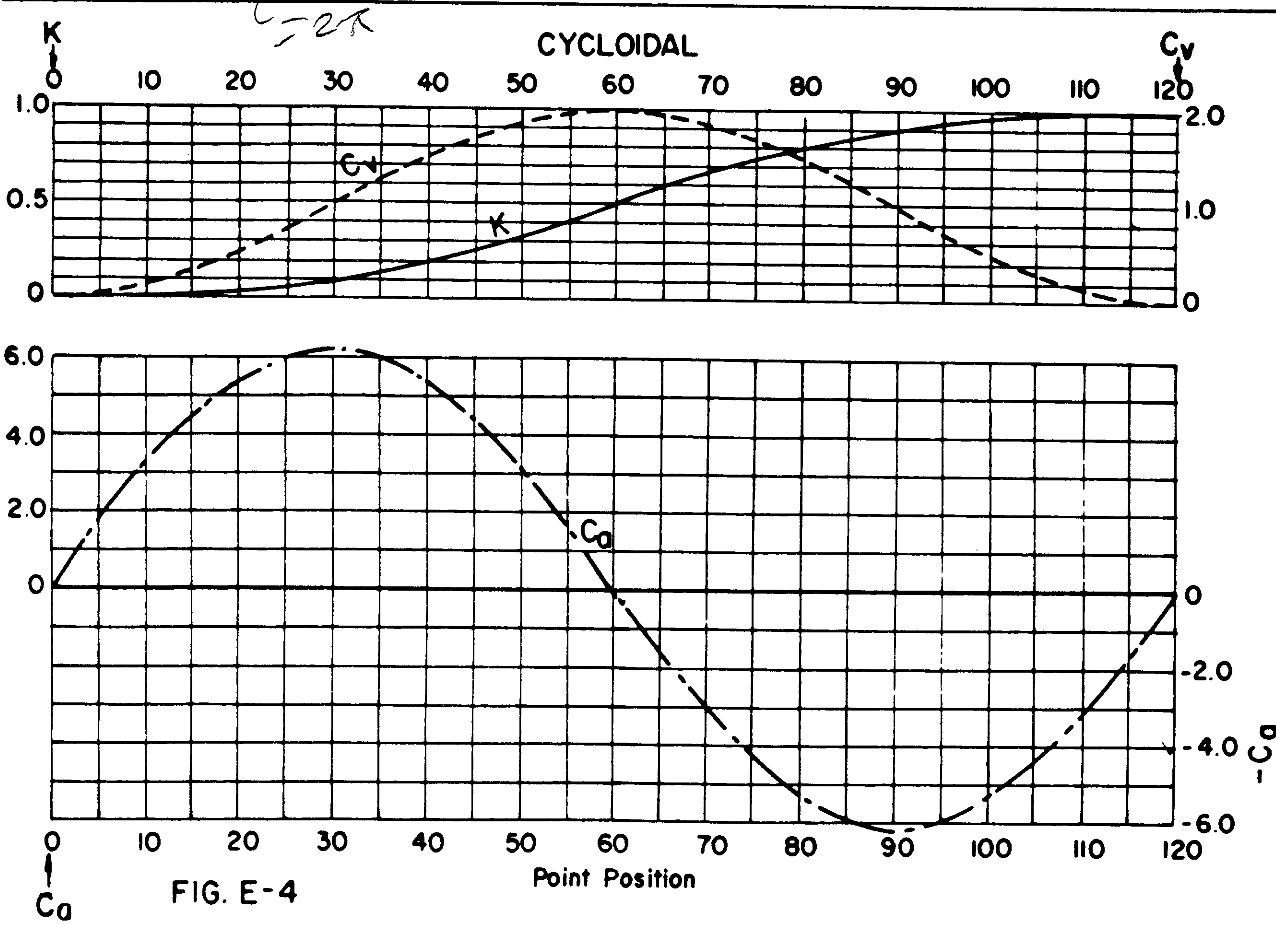


FIG. E-4

TABLE E-5. MODIFIED TRAPEZOID FACTORS

Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca
0	0.00000	0.0000	0.0000	30	0.10451	1.0000	4.8881	60	0.50000	2.0000	0.0000	90	0.89549	1.0000	-4.8881
1	0.00001	0.0021	0.5110	31	0.11300	1.0407	4.8881	61	0.51667	1.9979	-0.5110	91	0.90370	0.9593	-4.8881
2	0.00005	0.0085	1.0163	32	0.12181	1.0815	4.8881	62	0.53328	1.9915	-1.0163	92	0.91153	0.9185	-4.8881
3	0.00016	0.0190	1.5105	33	0.13100	1.1222	4.8881	63	0.54985	1.9810	-1.5105	93	0.91900	0.8778	-4.8881
4	0.00037	0.0336	1.9882	34	0.14053	1.1629	4.8881	64	0.56623	1.9664	-1.9882	94	0.92614	0.8371	-4.8881
5	0.00073	0.0521	2.4440	35	0.15036	1.2037	4.8881	65	0.58255	1.9479	-2.4440	95	0.93294	0.7963	-4.8881
6	0.00120	0.0743	2.8731	36	0.16057	1.2444	4.8881	66	0.59873	1.9257	-2.8731	96	0.93939	0.7556	-4.8881
7	0.00198	0.0999	3.2708	37	0.17113	1.2851	4.8881	67	0.61467	1.9001	-3.2708	97	0.94555	0.7149	-4.8881
8	0.00293	0.1287	3.6326	38	0.18203	1.3259	4.8881	68	0.63036	1.8713	-3.6326	98	0.95136	0.6741	-4.8881
9	0.00413	0.1604	3.9546	39	0.19323	1.3666	4.8881	69	0.64586	1.8396	-3.9546	99	0.95679	0.6334	-4.8881
10	0.00561	0.1945	4.2333	40	0.20471	1.4073	4.8881	70	0.66101	1.8055	-4.2333	100	0.96187	0.5927	-4.8881
11	0.00738	0.2308	4.4655	41	0.21669	1.4481	4.8881	71	0.67592	1.7693	-4.4655	101	0.96666	0.5519	-4.8881
12	0.00946	0.2688	4.6489	42	0.22891	1.4888	4.8881	72	0.69053	1.7312	-4.6489	102	0.97106	0.5512	-4.8881
13	0.01186	0.3081	4.7813	43	0.24147	1.5295	4.8881	73	0.70476	1.6919	-4.7813	103	0.97517	0.4705	-4.8881
14	0.01460	0.3483	4.8613	44	0.25443	1.5703	4.8881	74	0.71873	1.6517	-4.8613	104	0.97894	0.4297	-4.8881
15	0.01767	0.3890	4.8881	45	0.26767	1.6110	4.8881	75	0.73233	1.6110	-4.8881	105	0.98233	0.3890	-4.8881
16	0.02106	0.4297	4.8881	46	0.28126	1.6517	4.8613	76	0.74557	1.5703	-4.8881	106	0.98540	0.3483	-4.8613
17	0.02483	0.4705	4.8881	47	0.29524	1.6919	4.7813	77	0.75853	1.5295	-4.8881	107	0.98814	0.3081	-4.7813
18	0.02894	0.5112	4.8881	48	0.30947	1.7312	4.6489	78	0.77109	1.4888	-4.8881	108	0.99054	0.2688	-4.6489
19	0.03334	0.5519	4.8881	49	0.32408	1.7693	4.4655	79	0.78331	1.4481	-4.8881	109	0.99262	0.2308	-4.4655
20	0.03813	0.5927	4.8881	50	0.33899	1.8055	4.2333	80	0.79529	1.4073	-4.8881	110	0.99439	0.1945	-4.2333
21	0.04321	0.6334	4.8881	51	0.35414	1.8396	3.9546	81	0.80677	1.3666	-4.8881	111	0.99587	0.1604	-3.9546
22	0.04864	0.6741	4.8881	52	0.36964	1.8713	3.6326	82	0.81797	1.3259	-4.8881	112	0.99707	0.1287	-3.6326
23	0.05445	0.7149	4.8881	53	0.38533	1.9001	3.2708	83	0.82887	1.2851	-4.8881	113	0.99802	0.0999	-3.2708
24	0.06061	0.7556	4.8881	54	0.40127	1.9257	2.8731	84	0.83943	1.2444	-4.8881	114	0.99874	0.0743	-2.8731
25	0.06706	0.7963	4.8881	55	0.41745	1.9479	2.4440	85	0.84964	1.2037	-4.8881	115	0.99927	0.0521	-2.4440
26	0.07386	0.8371	4.8881	56	0.43377	1.9664	1.9882	86	0.85947	1.1629	-4.8881	116	0.99963	0.0336	-1.9882
27	0.08100	0.8778	4.8881	57	0.45015	1.9810	1.5105	87	0.86900	1.1222	-4.8881	117	0.99984	0.0190	-1.5105
28	0.08847	0.9185	4.8881	58	0.46672	1.9915	1.0163	88	0.87819	1.0815	-4.8881	118	0.99995	0.0085	-1.0163
29	0.09630	0.9593	4.8881	59	0.48333	1.9979	0.5110	89	0.88700	1.0407	-4.8881	119	0.99999	0.0021	-0.5110
30	0.10451	1.0000	4.8881	60	0.50000	2.0000	0.0000	90	0.89549	1.0000	-4.8881	120	1.00000	0.0000	0.0000

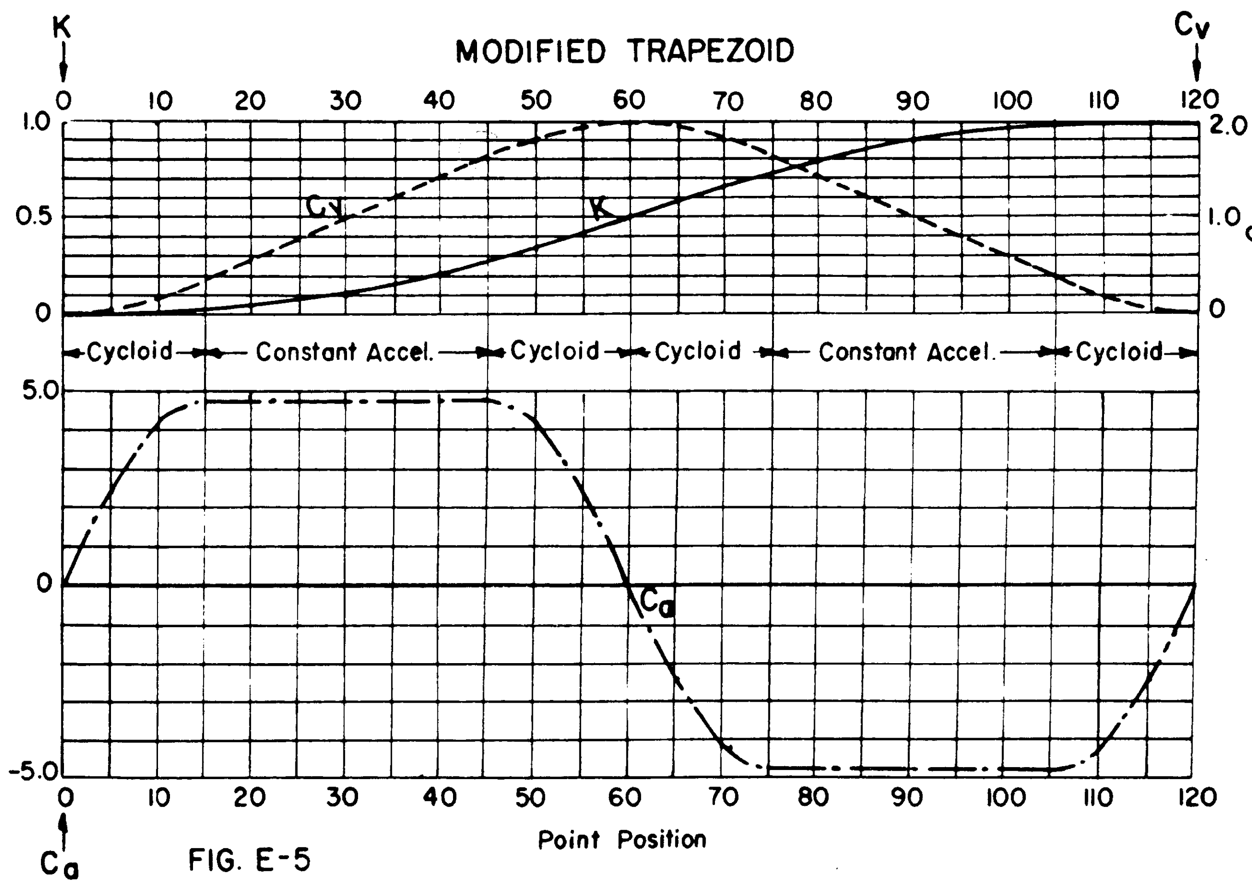


FIG. E-5

TABLE E-6. MODIFIED SINE FACTORS

Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca	Pt.	K	Cv	Ca
0	0.00000	0.0000	0.0000	30	0.11718	1.0997	4.7874	60	0.50000	1.7596	0.0000	90	0.88282	1.0997	-4.7874
1	0.00000	0.0024	0.5778	31	0.12650	1.1392	4.6380	61	0.51466	1.7588	-0.1929	91	0.89182	1.0594	-4.8809
2	0.00005	0.0096	1.1493	32	0.13616	1.1778	4.5829	62	0.52931	1.7564	-0.3856	92	0.90048	1.0184	-4.9685
3	0.00018	0.0215	1.7082	33	0.14613	1.2156	4.4722	63	0.54393	1.7523	-0.5778	93	0.90880	0.9767	-5.0501
4	0.00042	0.0380	2.2484	34	0.15642	1.2524	4.3561	64	0.55851	1.7467	-0.7693	94	0.91676	0.9343	-5.1255
5	0.00083	0.0589	2.7640	35	0.16701	1.2882	4.2347	65	0.57304	1.7395	-0.9599	95	0.92436	0.8912	-5.1946
6	0.00142	0.0840	3.2493	36	0.17788	1.3229	4.1081	66	0.58750	1.7307	-1.1493	96	0.93161	0.8477	-5.2574
7	0.00224	0.1130	3.6989	37	0.18905	1.3566	3.9765	67	0.60178	1.7204	-1.3373	97	0.93849	0.8036	-5.3138
8	0.00331	0.1455	4.1081	38	0.20049	1.3892	3.8401	68	0.61617	1.7084	-1.5237	98	0.94500	0.7592	-5.3638
9	0.00467	0.1813	4.4723	39	0.21220	1.4206	3.6989	69	0.63035	1.6950	-1.7082	99	0.95114	0.7143	-5.4072
10	0.00634	0.2199	4.7874	40	0.22417	1.4508	3.5553	70	0.64442	1.6800	-1.8907	100	0.95690	0.6691	-5.4440
11	0.00834	0.2610	5.0501	41	0.23638	1.4798	3.4034	71	0.65835	1.6635	-2.0708	101	0.96229	0.6236	-5.4742
12	0.01069	0.3040	5.2574	42	0.24883	1.5075	3.2493	72	0.67214	1.6455	-2.2484	102	0.96730	0.5778	-5.4977
13	0.01341	0.3484	5.4072	43	0.26150	1.5340	3.0912	73	0.68577	1.6260	-2.4233	103	0.97192	0.5320	-5.5145
14	0.01650	0.3939	5.4977	44	0.27439	1.5590	2.9294	74	0.69924	1.6051	-2.5952	104	0.97611	0.4860	-5.5246
15	0.01998	0.4399	5.5280	45	0.28748	1.5828	2.7640	75	0.71252	1.5828	-2.7640	105	0.98002	0.4399	-5.5280
16	0.02389	0.4860	5.5246	46	0.30076	1.6051	2.5952	76	0.72561	1.5590	-2.9294	106	0.98350	0.3939	-5.4977
17	0.02808	0.5320	5.5145	47	0.31423	1.6260	2.4233	77	0.73850	1.5340	-3.0912	107	0.98659	0.3484	-5.4072
18	0.03270	0.5778	5.4977	48	0.32786	1.6455	2.2484	78	0.75117	1.5075	-3.2493	108	0.98931	0.3040	-5.2574
19	0.03771	0.6236	5.4742	49	0.34165	1.6635	2.0708	79	0.76362	1.4798	-3.4034	109	0.99166	0.2610	-5.0501
20	0.04310	0.6691	5.4440	50	0.35558	1.6800	1.8907	80	0.77583	1.4508	-3.5553	110	0.99366	0.2199	-4.7874
21	0.04886	0.7143	5.4072	51	0.36965	1.6950	1.7082	81	0.78780	1.4206	-3.6989	111	0.99533	0.1813	-4.4722
22	0.05500	0.7592	5.3638	52	0.38383	1.7084	1.5237	82	0.79951	1.3892	-3.8401	112	0.99669	0.1455	-4.1081
23	0.06151	0.8036	5.3138	53	0 39812	1.7204	1.3373	83	0.81095	1.3566	-3.9765	113	0.99776	0.1130	-3.6989
24	0.06839	0.8477	5.2574	54	0.41250	1.7307	1.1493	84	0.82212	1.3229	-4.1081	114	0.99858	0.0840	-3.2493
25	0.07564	0.8912	5.1946	55	0.42696	1.7395	0.9599	85	0.83299	1.2882	-4.2347	115	0.99917	0.0589	-2.7640
26	0.08324	0.9343	5.1255	56	0.44149	1.7467	0 7693	86	0.84358	1.2524	-4.3561	116	0.99958	0.0380	-2.2484
27	0.09120	0.9767	5.0501	57	0.45607	1.7523	0.5778	87	0.85387	1.2156	-4.4722	117	0.99982	0.0215	-1.7082
28	0.09952	1.0184	4.9685	58	0.47069	1.7564	0.3856	88	0.86384	1.1778	-4.5829	118	0.99995	0.0096	-1.1493
29	0.10818	1.0594	4.8809	59	0.48534	1.7588	0.1929	89	0.87350	1.1392	-4.6880	119	1.00000	0.0024	-0.5778
30	0.11718	1.0997	4.7874	60	0.50000	1.7596	0 0000	90	0.88282	1.0997	-4.7874	120	1.00000	0.0000	0.0000

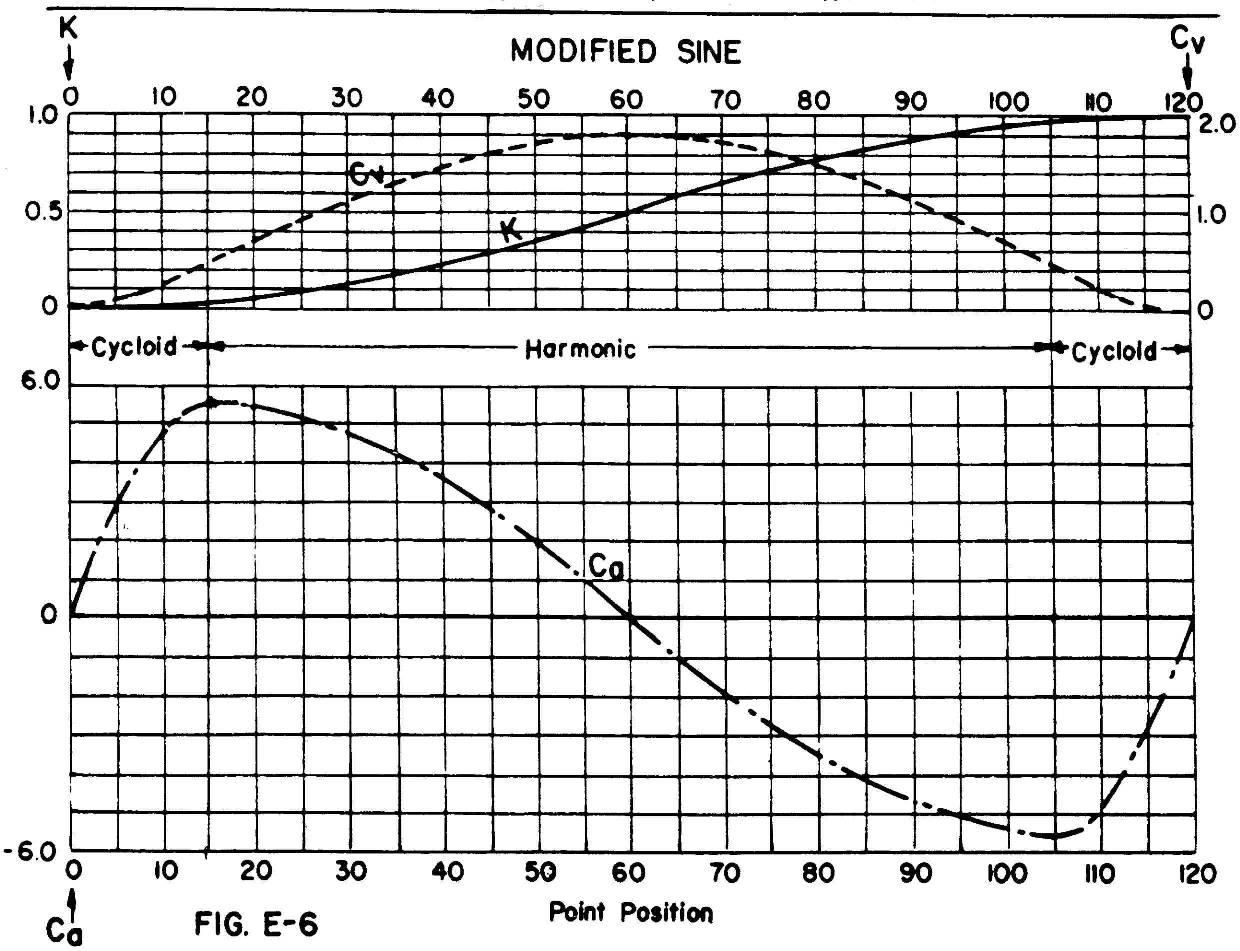


FIG. E-6

SECTION F

Cam Size Determination

Basic Considerations. Usually the first consideration in designing a cam is its physical size. A cam of minimum size may be desired, or a cam of a fixed size may have to be evaluated. The determining factors for a cam of minimum size are the maximum pressure angle, the least radius of curvature, and the cam shaft diameter.

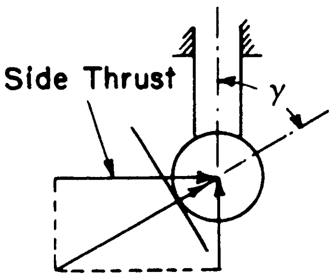


FIG. F-1

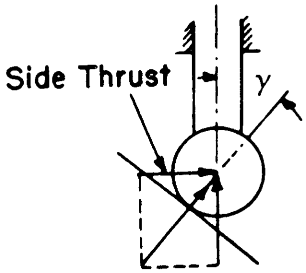


FIG. F-2

Pressure Angles. As may be seen in Figs. F-1 and F-2, the greater the pressure angle, the greater the side thrust on the follower. Too much side thrust may result in jamming the follower rod in its guides. For this reason, it is customary to limit the maximum pressure angle to approximately 30 degrees for cams with translating followers. If the follower is on center, pressure angles may be easily determined mathematically. The mathematics for an off-center or swinging arm follower are more complex, but pressure angles may be found by inspection from a layout, or by graphical methods which will be described later. Pressure angles depend on the velocities of the cam and follower. From Fig. F-3, the tangent of the pressure angle equals the quotient of the follower velocity and the cam velocity.

$$\tan \gamma = \frac{V_f}{V_c} \qquad (1)$$

Converting this equation to terms of displacements of follower and cam, this becomes

$$\tan \gamma = \frac{57.3\, C_v h}{R_n \beta} \qquad (2)$$

where R_n = radius to the reference point on the pitch curve, and the other symbols as in section E nomenclature. This equation is valid at any point on the pitch curve with an on-center translating follower.

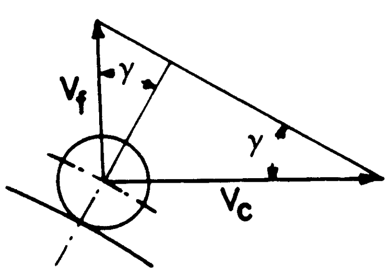


FIG. F-3

Pitch Radius. To determine the pitch radius of a cam of minimum size, the following equations are convenient:

For a cycloidal, constant acceleration or modified trapezoid curve,

$$R_p = \frac{200h}{\beta} \qquad (3)$$

For a harmonic curve,

$$R_p = \frac{157h}{\beta} \qquad (4)$$

For a modified sine curve,

$$R_p = \frac{176h}{\beta} \qquad (5)$$

These equations give the pitch radius of a cam with a maximum pressure angle slightly less than 30 degrees.

Radius of Curvature If, as shown in Figs. F-4 and F-5, the radius of curvature (R_c) is held constant and the radius of follower (r_f) increased, the cam profile will eventually become undercut, and the follower will not follow the prescribed motion. To prevent this, the least radius of curvature must be substantially greater than the radius of the follower. A rigorous calculation of the radius of curvature is quite involved. However, the following method gives sufficiently accurate results for an on-center translating motion. Graphical methods for all types are described later.

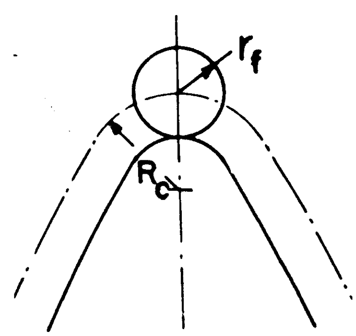


FIG. F-4

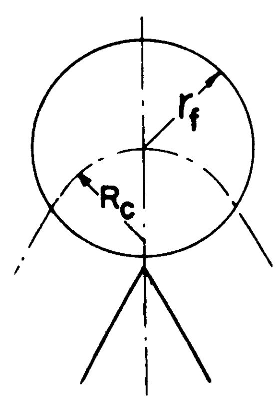


FIG. F-5

In Fig. F-6, for a very small section AB of a cam pitch curve, it is reasonable to assume that the center of the radius of curvature lies very near the intersection of normal lines AP and BP. It is also reasonable to assume that, for the small section involved, arcs AB and AD are substantially equal. With these assumptions:

$$R_A \sin \Delta\theta = R_c \sin \alpha$$

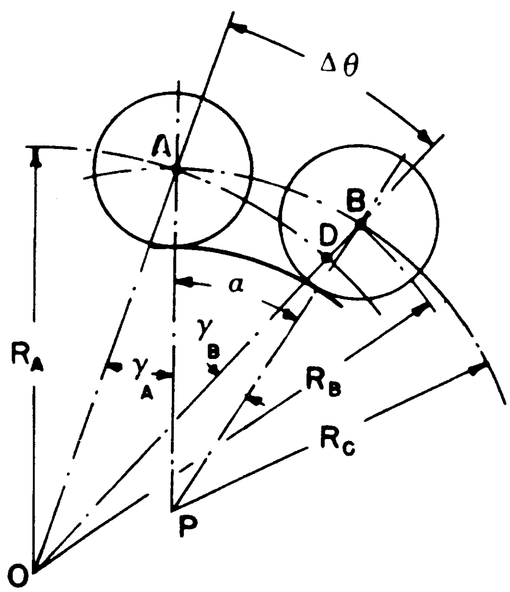


FIG. F-6

From the geometry of the layout

$$\alpha = \gamma_A - \gamma_B + \Delta\theta$$

where γ_A and γ_B are the pressure angles at reference points A and B. Therefore,

$$R_c = R_A \frac{\sin \Delta\theta}{\sin \alpha} \qquad (6)$$

If α is positive, the center of R_c lies inside the pitch curve periphery; if α is negative, the center lies outside the periphery. This method gives good accuracy near the terminals of the curve, where the increment BD is quite small. Accuracy decreases as the transition point is approached. In this area, more nearly accurate results may be obtained by a better approximation of arc AB.

$$\text{AD} = R_A \sin \Delta\theta$$

$$\text{BD} = R_B - R_A$$

$$\text{AB} = \sqrt{\overline{\text{AD}}^2 + \overline{\text{BD}}^2}$$

$$R_c = \frac{\text{AB}}{\sin \alpha} \qquad (7)$$

The least radius of curvature occurs at or near the point of maximum negative acceleration. Calculation of the radius at this point provides an evaluation of the cam size as compared with the diameter of the follower.

Cam Shaft Diameter. Usually before the cam has been designed the cam shaft diameter has been determined from stress and deflection factors. The cam must have a hub of sufficient size to accommodate this shaft and its key. The base radius of an open cam profile must be greater than the hub radius. This is also necessary on closed cams where the hub is on the grooved side.

Graphical Methods. Procedures for graphically determining pressure angles, pitch radii, and radii of curvature are described under Figs. F-7 through F-12. The methods are as accurate as the draftsmanship.

Pressure Angles—Nomenclature

- h = total linear displacement of follower (in.)
- b = length of swinging arm (in.)
- r = radius—cam axis to reference point (in.)
- x = offset of follower center from cam axis (in.)
- C_v = coefficient of velocity at reference point
- ϕ = total angular displacement of swinging arm (deg.)
- β = total angular displacement of cam (deg.)
- γ = cam pressure angle (deg.)

On-Center Translating Follower. See Fig. F-7.

To determine the pressure angle at any reference point A when linear and angular displacements and radius are known,

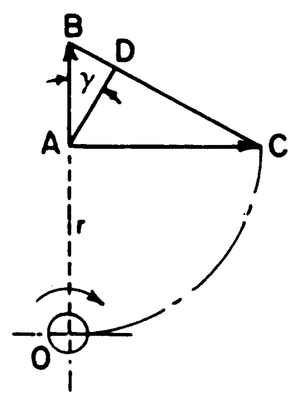


FIG. F-7

1. Draw AB equal to $57.3C_vh/\beta$ in direction of follower motion.
2. Draw AC equal and perpendicular to radius r(OA) in direction of cam rotation.
3. Draw BC to complete triangle ABC.
4. Draw AD perpendicular to BC.
5. Measure pressure angle γ.

To determine radius to the pitch curve for desired pressure angle at any reference point A when linear and angular displacements are known,

1. Draw AB equal to $57.3C_vh/\beta$ in direction of follower motion.
2. Draw AD, indefinite in length, at desired angle γ with AB so that BD will be in direction of cam rotation.
3. Draw BC, indefinite in length, perpendicular to AD.
4. Draw AC perpendicular to AB in direction of cam rotation.
5. Scale AC, which is the required radius.

Offset Translating Follower. See Fig. F-8(a).

To determine pressure angle at any reference point A when linear and angular displacements and radius are known,

1. Draw AB equal to $57.3C_v h/\beta$ in direction of follower motion.
2. Draw AC equal and perpendicular to radius r(OA) in direction of cam rotation.
3. Draw BC, completing triangle ABC.
4. Draw AD perpendicular to BC.
5. Measure pressure angle γ.

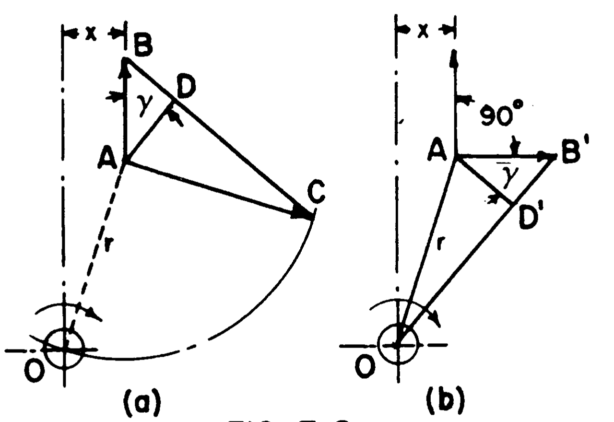


FIG. F-8

To determine radius to pitch curve for desired pressure angle at any reference point A, when linear and angular displacements are known, Fig. F-8(b),

1. Draw AB′ equal to $57.3C_v h/\beta$, turned 90° from line of follower motion, and in direction of cam rotation.
2. Draw AD′, indefinite in length, at desired angle γ with AB′.
3. Draw B′O, perpendicular to AD′, intersecting vertical cam axis at O.
4. Scale OA, which is the required radius.

Swinging Arm Follower. See Fig. 9.

To determine pressure angle at any reference point A when angular displacements of arm and cam, radius, arm length, and relative position of cam axis O and fulcrum Q are known, Fig. 9(a),

1. Draw AB equal to $C_v b\phi/\beta$ perpendicular to QA in direction of follower motion.
2. Draw AC equal and perpendicular to OA in direction of cam rotation.
3. Draw BC completing triangle ABC.
4. Draw AD perpendicular to BC.
5. Measure pressure angle γ.

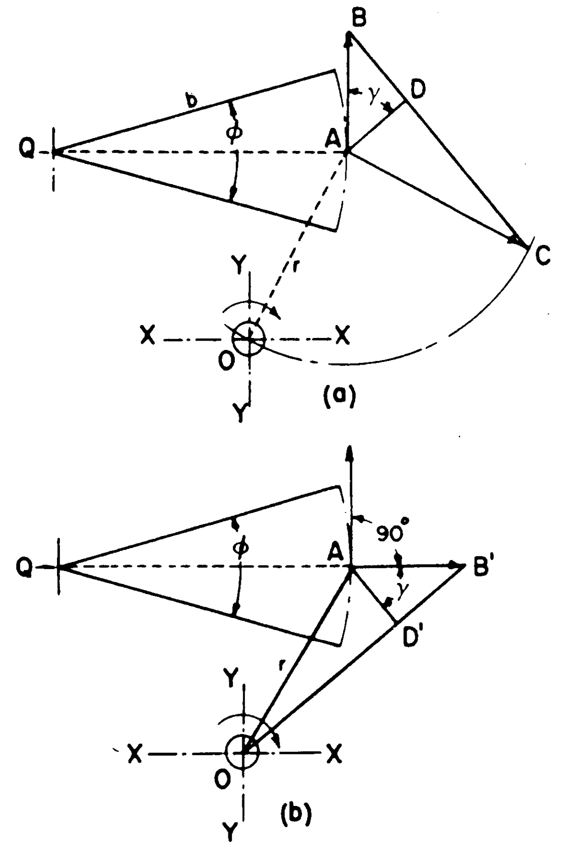


FIG. F-9

To determine radius to pitch curve for desired pressure angle when angular displacement of arm and cam, arm length, and relative position of cam axis O and fulcrum Q are known, Fig. 9(b),

CASE I. When relation of fulcrum Q to either the horizontal axis X-X, or the vertical axis Y-Y is known, Fig. F-9(b).

1. Draw AB′ equal to $C_v b\phi/\beta$, turned 90° from line of follower motion, and in direction of cam rotation.
2. Draw AD′, indefinite in length, at desired angle γ with AB′.
3. Draw B′O, perpendicular to AD′ intersecting axis X-X or Y-Y at O.
4. Scale OA, which is the required radius.

CASE II. When relation of fulcrum Q to cam axis is not fixed, Fig. 9(a),

1. Draw AB equal to $C_v b\phi/\beta$ perpendicular to QA in direction of follower motion.
2. Draw AD, indefinite in length, at desired angle γ with AB, so that BD will be in direction of cam rotation.
3. Draw BC, indefinite in length, perpendicular to AD.
4. With A as center, inscribe any radius, intersecting BC at C. (Suggested radius approximately equal to QA.)
5. Draw AO equal and perpendicular to AC, thus locating cam axis O.

Radius of Curvature—Nomenclature

h = total linear displacement of follower (in.)
b = length of swinging arm (in.)
C_v = coefficient of velocity at reference point.
C_a = coefficient of acceleration at reference point
v_f = velocity of follower at one radian per second cam shaft speed (in./sec.)
a_f = acceleration of follower at one radian per second cam shaft speed (in./sec.^{2})
a_n = normal acceleration of swinging arm at one radian per second cam shaft speed (in./sec.^{2})
x = offset of follower center from cam axis (in.)
ϕ = total angular displacement of swinging arm (deg.)
β = total angular displacement of cam (deg.)

On-Center Translating Follower. See Fig. F-10(a) and (b).

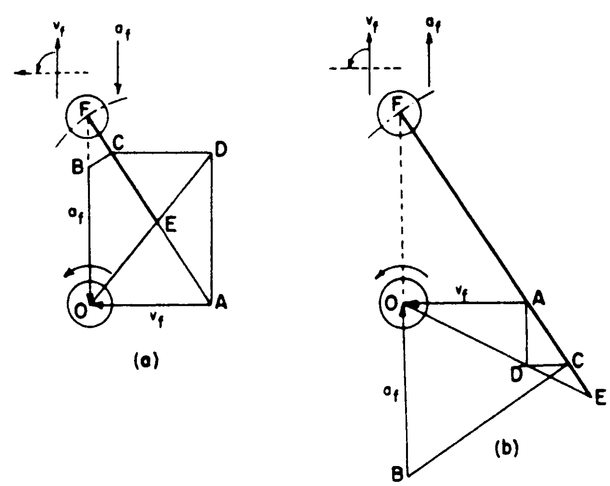


FIG. F-10

To determine radius of curvature at any point F,

1. Draw OA equal to $v_f = 57.3C_v h/\beta$, turned 90° in direction of cam rotation, directed toward O.
2. Draw OB equal to $a_f = C_a h(57.3/\beta)^2$, directed toward O.
3. Draw BC perpendicular to AF.
4. Draw CD, indefinite in length, parallel to OA.
5. Draw AD, parallel to OF, interesecting CD at D.
6. Draw OD, intersecting AF at E.
7. Scale EF, which is the radius of curvature.

Offset Translating Follower. See Fig. F-11(a) and (b).

To determine radius of curvature at any point F, proceed exactly as in the case of the on-center follower.

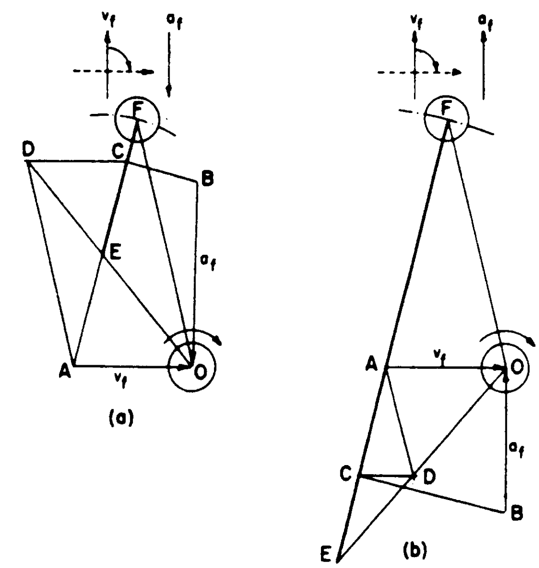


FIG. F-11

Swinging Arm Follower. See Fig. F-12.

To determine the radius of curvature at any point F,

1. Draw OA equal to $v_f = C_v b\phi/\beta$, turned 90° in direction of cam rotation directed toward O.
2. Draw FH, equal to OA, intersecting circle QHF at H.
3. Draw HG perpendicular to QF.
4. Draw OJ, equal to a_n = GF, parallel to QF, directed in sense of F to Q.
5. Draw JB equal to $a_f = 57.3C_a b\phi/\beta^2$, directed toward J.
6. Draw BC perpendicular to AF.
7. Draw CD, indefinite in length, parallel to OA.
8. Draw AD, parallel to OF, intersecting CD at D.
9. Draw OD, intersecting AF at E.
10. Scale EF, which is the radius of curvature.

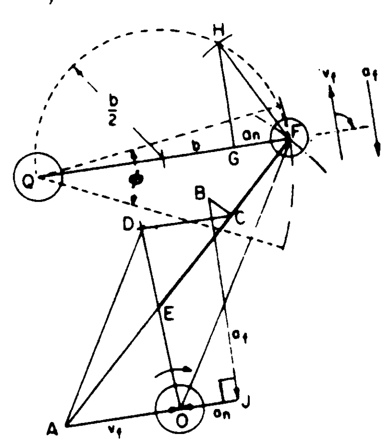
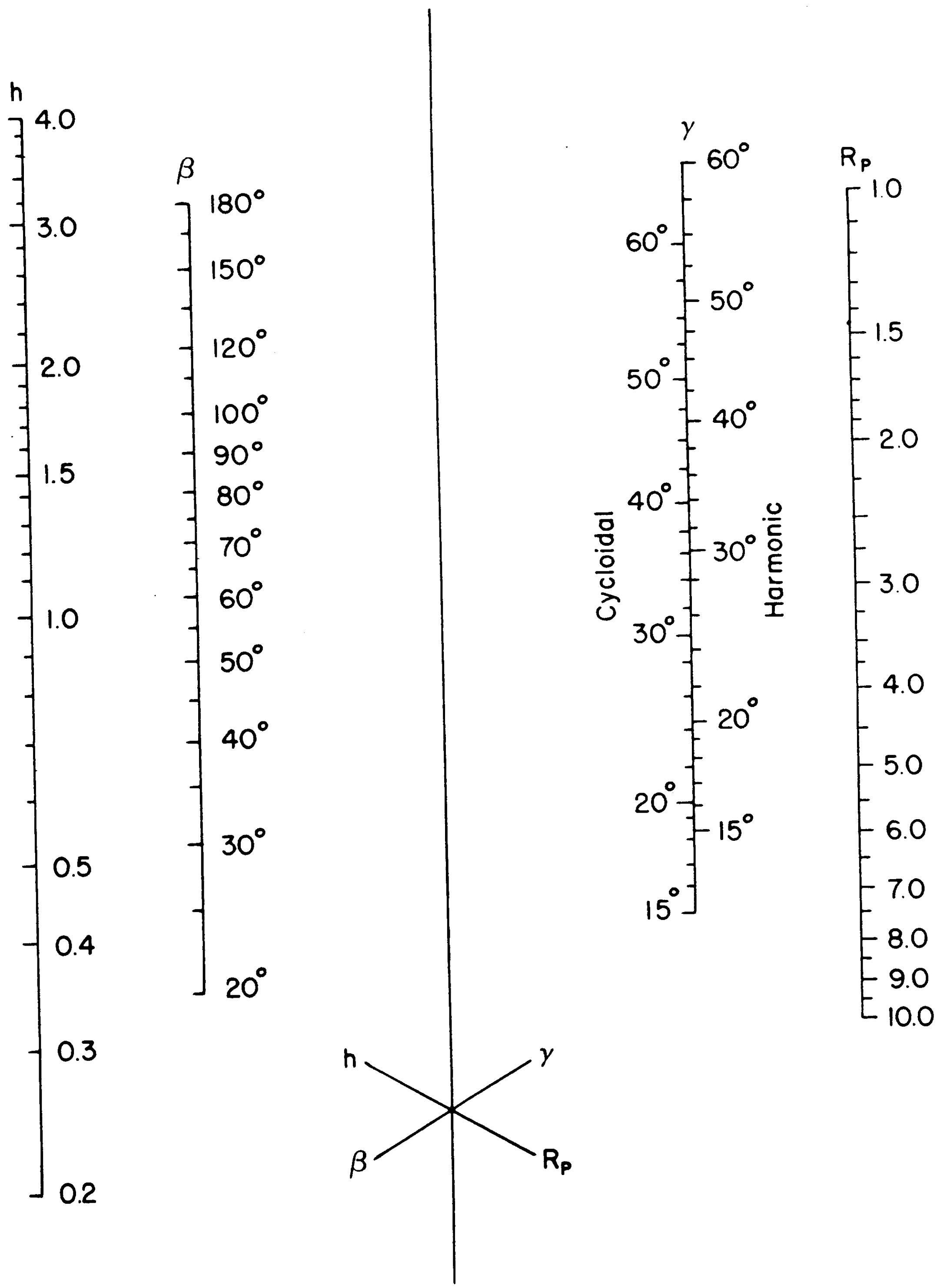


FIG. F-12

References

1. Cams—Design, Dynamics and Accuracy. H. A. Rothbart. John Wiley & Sons, Inc. 1956.
2. Cam Pressure Angles. R. T. Hinkle. Machine Design. July 1955.
3. Disc Cam Curvature. J. Hirschorn. Fifth Mechanism Conference Transactions. 1958.

NOMOGRAPH FOR MAXIMUM PRESSURE ANGLE



SECTION G

Force Analysis

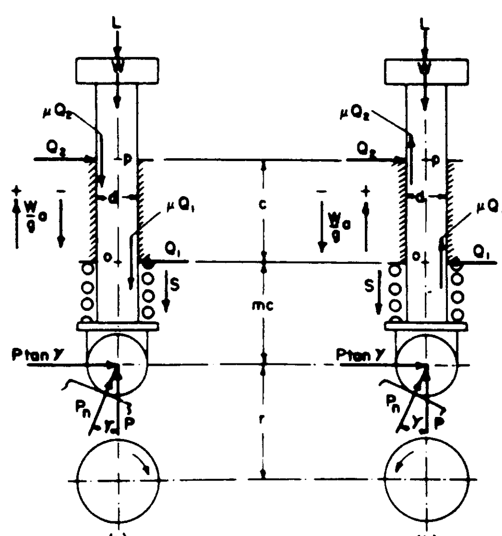
Nomenclature

a	=	follower acceleration (in./sec.^{2})
c	=	length of cam rod guide (in.)
d	=	cam rod diameter (in.)
f	=	friction factor
g	=	gravity constant (386 in./sec.^{2})
m	=	ratio of follower overhang to guide length
r	=	radius to reference point (in.)
P	=	force parallel to cam rod (lbs.)
P_n	=	force normal to cam profile (lbs.)
Q_1, Q_2	=	forces normal to cam rod (lbs.)
L	=	external force (lbs.)
S	=	spring force (lbs.)
T	=	torque (lb.-in.)
W	=	weight of accelerated elements (lbs.)
γ	=	cam pressure angle (deg.)
μ	=	coefficient of friction

Cam Forces. The forces acting on a cam include the inertial force, weight of elements, external loads, spring forces, and friction loads. Neglecting the friction between the roller follower and the cam profile, Fig. G-1 shows the forces applied to an open cam. In order to keep the same direction for the forces normal to the cam rod, the direction of rotation in Fig. G-1(b) is shown opposite to that of Fig. G-1(a). The forces acting on this system are to be determined. According to custom, forces acting to the left or upward are considered positive; those acting to the right or downward, negative. Also, counterclockwise moments are positive; clockwise moments are negative.

To maintain equilibrium, the sum of all horizontal forces must equal zero.

$$\Sigma F_h = 0 = Q_1 - Q_2 - P \tan \gamma \tag{1}$$



(a) FIG. G-1 (b)

Also, from Newton's laws, the sum of all forces in the direction of acceleration equals the inertial force.

$$\Sigma F_v = \pm \frac{W}{g} a = P - W - L - S \mp (\mu Q_1 + \mu Q_2) \tag{2}$$

From this,

$$P = \pm \frac{W}{g} a + W + L + S \pm (\mu Q_1 + \mu Q_2) \tag{3}$$

Also, the sum of the moments about any point in the system should equal zero. To evaluate Q_1 and Q_2, moments are taken about points "o" and "p."

$$\Sigma M_o = 0 = (P \tan \gamma) mc - Q_2 c \mp 0.5 \mu Q_1 d \pm 0.5 \mu Q_2 d \tag{4}$$

$$\Sigma M_p = 0 = (P \tan \gamma)(mc + c) - Q_1 c \mp 0.5 \mu Q_1 d \pm 0.5 \mu Q_2 d \tag{5}$$

Considering the friction moments as negligible,

$$(P \tan \gamma) mc - Q_2 c = 0 \tag{6}$$

$$(P \tan \gamma)(mc + c) - Q_1 c = 0 \tag{7}$$

and

$$Q_2 = (P \tan \gamma) m \tag{8}$$

$$Q_1 = (P \tan \gamma)(m + 1) \tag{9}$$

Adding Equations 8 and 9 and substituting in Equation 3, and solving for P,

$$P = \frac{\pm (W/g) a + W + L + S}{1 \pm \mu (2m + 1) \tan \gamma} \tag{10}$$

In Equation 10, the expression $\mu(2m + 1) \tan \gamma$ represents the effect of friction. Note that it depends on μ, m, and γ, and if it is denoted by f, the equation becomes

$$P = \frac{\pm (W/g) a + W + L + S}{1 \pm f} \tag{11}$$

Taking "μ" as an average of 0.10, Table G-1 shows the magnitude of f for various values of m and γ.

γ \ m	1/8	1/6	1/4	1/2	1	2
10°	0.022	0.023	0.026	0.035	0.053	0.088
15	0.033	0.036	0.040	0.054	0.080	0.134
20	0.045	0.049	0.055	0.073	0.109	0.182
25	0.058	0.062	0.070	0.093	0.140	0.233
30	0.072	0.077	0.087	0.115	0.173	0.288
35	0.087	0.093	0.105	0.140	0.210	0.350
40	0.105	0.112	0.126	0.168	0.252	0.419
45	0.125	0.133	0.150	0.200	0.300	0.500

TABLE G-1 Friction Factors (f)

Normal Force. This force determines the contact stress between cam and roller. It is found by the following equation,

$$P_n = \frac{P}{\cos \gamma} \tag{12}$$

Torque. The torque on the cam shaft is determined by

$$T = rP \tan \gamma \tag{13}$$

The maximum torque determines the cam shaft load, the power to drive the system and the size of the drive.

All of the preceding applies to an open cam with a spring constrained follower. Note that P must always be directed upward to maintain contact between the cam and follower. The spring must be strong enough to accomplish this condition. The method of determining the necessary spring specifications is described later.

Closed Cam Forces. No spring is required with a closed or grooved cam. The direction of P may be downward, contact then being between the outer profile and the roller. Typical forces on a closed cam are shown in Fig. G-2.

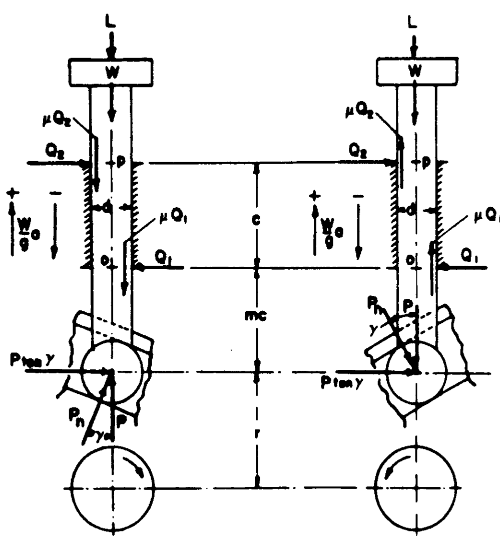


FIG. G-2

Spring Specifications. The following example demonstrates the procedure for determining the spring required for an open cam with spring constrained follower.

Example. Consider an open cam with an on-center translating follower, lifting a weight (W) of 20 lbs. through a displacement (h) of 0.75 in. The cam shaft speed is 120 RPM; the action takes place in 60° of cam rotation, and the cam curve is cycloidal. The friction forces are negligible.

The peak positive and negative accelerations occur at 15° and 45° respectively, where

$$a = \pm 6.28\,(0.75)\left(\frac{6 \times 120}{60}\right)^2 = \pm 679 \text{ in./sec.}^2$$

The peak inertial forces are

$$\frac{W}{g}\,a = \frac{20}{386}(\pm 679) = \pm 35 \text{ lbs.}$$

The weight and inertial forces are plotted in Fig. G-3. At B these forces total 55 lbs.; at D, −15 lbs. As the forces (P) must be positive at all points, the spring must exert a force greater than 15 lbs. at point D. Assume that an initial spring force of 5 lbs. is adequate at point A, and a total force of 10 lbs. is sufficient at point D. Then, at D, the spring must exert a force of 10 − (−15) or 25 lbs. The displacement of the cam at D = 0.90915 (0.75) = 0.6818 in. The spring force has increased from 5 lbs. to 25 lbs. or 20 lbs. Therefore the spring rate is

$$\frac{20}{0.6818} = 29.3 \text{ lbs. per in.}$$

The initial deflection of the spring is 5/29.3 = 0.170 in.

The deflection at B = 0.09085 (0.75) + 0.170 = 0.238 in. The spring force at B = 0.238 (29.3) = 7 lbs. The deflection at C = 0.5 (0.75) + 0.170 = 0.545 in. The spring force at C = 0.545 (29.3) = 16 lbs. The deflection at E = 0.750 + 0.170 = 0.920 in. The spring force at E = 0.92 (29.3) = 27 lbs. These forces are plotted in Fig. G-3 to complete the force diagram.

Assuming that the spring will come within a quarter inch of solid, the specifications for determining the spring and wire diameters are

Free length = 0.17 + 0.75 + 0.25 = 1.17 in.
Initial deflection = 0.17 in.
Working deflection = 0.17 + 0.75 = 0.92 in.
Spring rate = 29.3 lbs./in.

If a spring of suitable mean and wire diameters cannot be produced from these values, new assumptions must be made for the spring forces at A and D. Also, if machine speed were increased, a new set of values would result and new spring specifications would have to be determined.

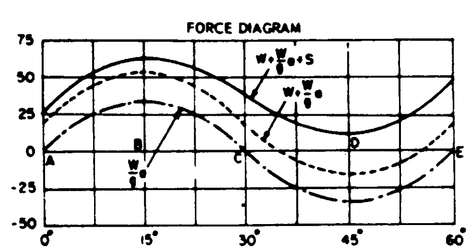


FIG G-3

Contact Stress

Nomenclature

C = radius factor
E_c = modulus of elasticity of cam material
E_f = modulus of elasticity of follower material
M = material factor
P_n = force normal to cam profile (lbs.)
R_c = radius of curvature of pitch curve (in.)
S_c = contact stress (psi)
L = width of cam face or follower (in.)
r_f = radius of roller follower (in.)

Equations. The compressive stress between two elastic bodies in contact under load (Fig. H-1) may be calculated from the Hertz equation, which, applied to cam profile and roller follower is

$$S_c^2 = \frac{0.35P_n\left(\frac{1}{R_c \pm r_f} \mp \frac{1}{r_f}\right)}{L\left(\frac{1}{E_c} + \frac{1}{E_f}\right)} \qquad (1)$$

This may be simplified to

$$S_c = (10)^3\sqrt{\frac{P_nC}{LM}} \qquad (2)$$

where

$$M = \frac{E_c + E_f}{0.35E_cE_f}(10)^6$$

FLO $C = \dfrac{R_c}{(R_c - r_f)r_f}$ for a convex surface

F70 $C = \dfrac{R_c}{(R_c + r_f)r_f}$ for a concave surface

$C = \dfrac{1}{r_f}$ for a flat surface

If the radius of curvature (R_c) and the normal force (P_n) are of the same sign (positive or negative), the surface is convex; if they are of opposite signs, the surface is concave. If R_c is infinite, the surface is flat.

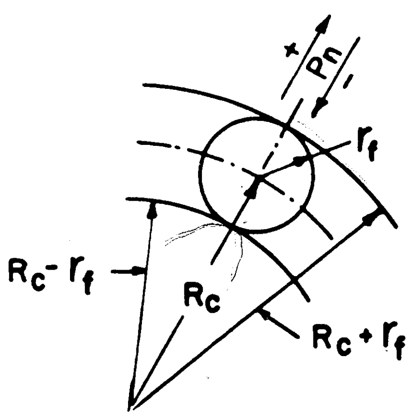


FIG. H-1

Material Factors. In Table H-1 are shown values of M for various cam and follower materials.

CAM	FOLLOWER	M
Steel	Steel	0.190
GM Meehanite	Steel	0.219
GA Meehanite	Steel	0.231
GB Meehanite	Steel	0.246
GM Meehanite	GA Meehanite	0.272
GM Meehanite	Ampco 18	0.292
GA Meehanite	Ampco 18	0.304
Ampco 20	Ampco 18	0.359

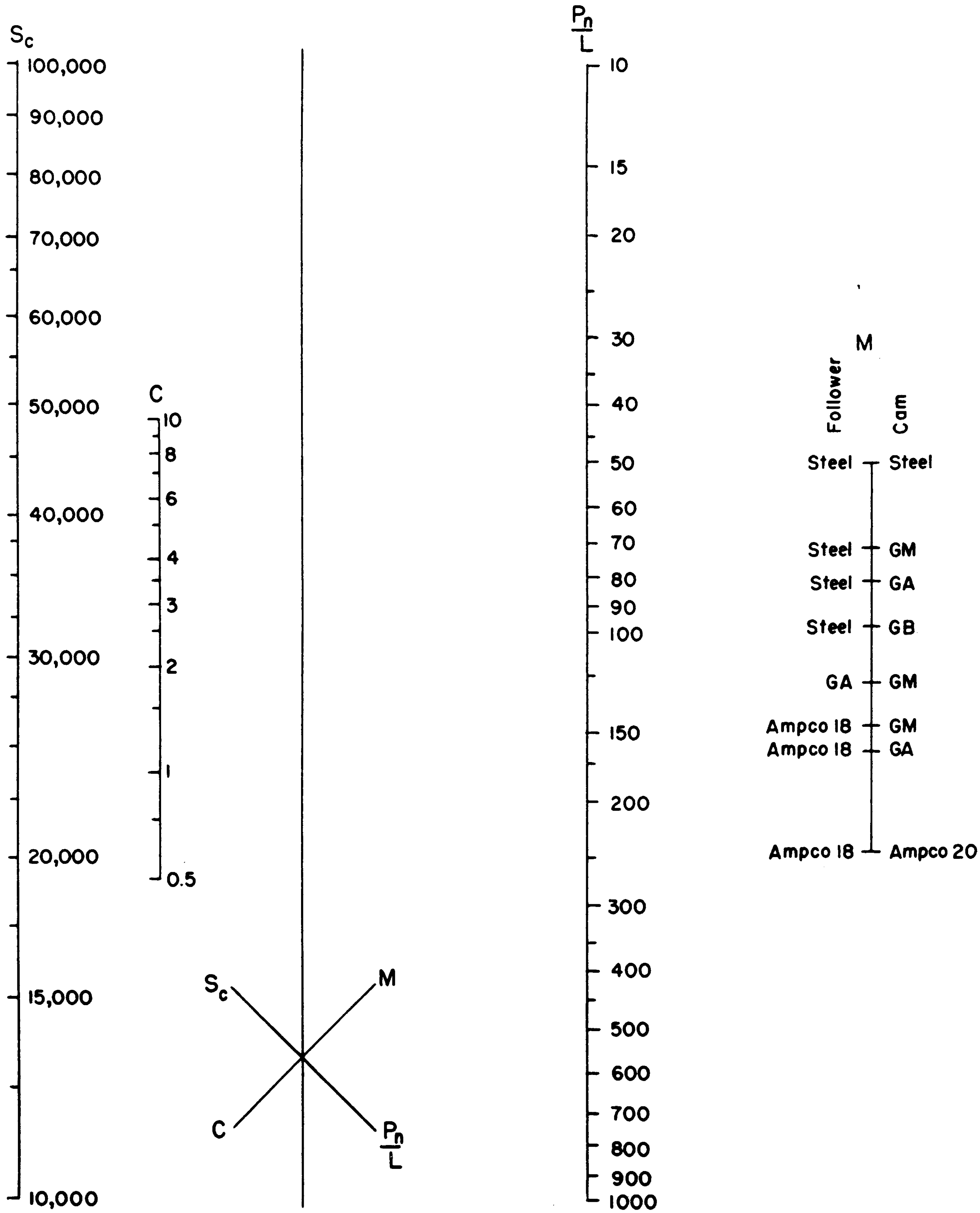
TABLE H-1

Design Stress. This should be limited to the following values:

Steel	100,000 psi
GM Meehanite	65,000 psi
GA Meehanite	55,000 psi
GB Meehanite	53,000 psi
Ampco 18	34,000 psi
Ampco 20	45,000 psi

If shock loads or excessive sliding are present, these values should be reduced by 25 to 40 per cent. For other materials, the design stress should not exceed one-third the ultimate compressive stress. Deflection of the cam shaft, or misalignment of the roller follower can greatly increase the contact stress. The shaft should be of ample size and well supported. Care should be taken to assure good alignment of cam and follower.

NOMOGRAPH FOR CONTACT STRESS



SECTION J

Cam Design Summary

Design Procedure. Assuming that the linear displacement (h), the angular displacement (β), the weight (W) of the accelerated system, and the type of curve have been determined, and that the follower dimensions, cam and follower materials have been tentatively selected, the following will provide an orderly design procedure:

1. Determine the pitch radius (R_p) for an acceptable maximum pressure angle (Section F; Equations 3, 4, or 5).
2. Determine the minor radius (R_o) of the pitch curve.

$$R_o = R_p - 0.5h$$

3. Determine the displacement (y) at critical points (Section E; Equation 1). In general, these are the points of maximum positive and negative acceleration, adjacent points either preceding or following these peaks, and the midpoint. For instance, for a cycloidal curve, they are points 30, 31, 60, 90, and 91; for a harmonic curve, points 1, 60, 119.
4. Determine radius (R_n) at the critical points.

$$R_n = R_o + y$$

5. Determine pressure angles (γ) at critical points (Section F; Equation 2).
6. Determine radius of curvature (R_c) at the maximum positive and negative acceleration points (Section F; Equations 6 or 7). At this stage the follower diameter can be evaluated.
7. Determine maximum positive and negative accelerations (Section E; Equation 3).
8. Determine inertial force (Wa/g) at the peak acceleration points (Section A; Equation 16).
9. Determine force (P) acting on the system at the peak acceleration points (Section G; Equation 11).
10. Determine force (P_n) normal to the cam profile at the peak acceleration points (Section G; Equation 12).
11. Determine contact stress at the peak acceleration points (Section H; Equation 2).

Design Example. A closed or grooved cam, with an on-center translating roller follower, is to lift a weight of 50 lbs. through 1.5 in. in 75° of rotation at 150 RPM. The cam curve is to be cycloidal. The follower is 1.5 in. diameter and 1 in. wide. The cam is to be GM Meehanite and the follower, hardened steel. Friction is negligible.

Design Calculations. Given $h = 1.5$; $\beta = 75°$; $N = 150$; $r_f = 0.75$ and $L = 1.0$.

1. $R_p = \dfrac{200(1.5)}{75} = 4.0$

2. $R_o = 4.0 - 0.5(1.5) = 3.25$

3. From Table E-4
$y_{30} = 0.09085\ (1.5) = 0.136$
$y_{31} = 0.09940\ (1.5) = 0.149$
$y_{60} = 0.50000\ (1.5) = 0.750$
$y_{90} = 0.90915\ (1.5) = 1.363$
$y_{91} = 0.91726\ (1.5) = 1.376$

4. $R_{30} = 3.25 + 0.136 = 3.386$
$R_{31} = 3.25 + 0.149 = 3.399$
$R_{60} = 3.25 + 0.750 = 4.000$
$R_{90} = 3.25 + 1.363 = 4.613$
$R_{91} = 3.25 + 1.376 = 4.626$

5. $\text{Tan}\ \gamma_{30} = \dfrac{57.3\ (1.0)(1.5)}{3.386\ (75)} = 0.3384$
$\gamma_{30} = 18.70°$
$\text{Tan}\ \gamma_{31} = \dfrac{57.3\ (1.0523)(1.5)}{3.399\ (75)} = 0.3540$
$\gamma_{31} = 19.50°$
$\text{Tan}\ \gamma_{60} = \dfrac{57.3\ (2.0)(1.5)}{4.0(75)} = 0.5730$
$\gamma_{60} = 29.86°$
$\text{Tan}\ \gamma_{90} = \dfrac{57.3\ (1.0)(1.5)}{4.613\ (75)} = 0.2484$
$\gamma_{90} = 13.95°$
$\text{Tan}\ \gamma_{91} = \dfrac{57.3\ (0.9477)(1.5)}{4.626\ (75)} = 0.2348$
$\gamma_{91} = 13.21°$

6. $\Delta\theta = \dfrac{75}{120} = 0.62°$
$\alpha_{30} = 18.70 - 19.50 + 0.62 = -0.18°$
$R_c = 3.386\ \dfrac{\sin 0.62°}{-\sin 0.18°} = -11.8$ in.
$\alpha_{90} = 13.95 - 13.21 + 0.62 = 1.36°$
$R_c = 4.613\ \dfrac{\sin 0.62°}{\sin 1.36°} = 2.1$ in.

7. $a\ (max.) = \pm 6.2832\ (1.5)\left(\dfrac{6 \times 150}{75}\right)^2 =$
± 1360 in./sec.^{2}

8. $\dfrac{W}{g}\,a = \dfrac{50}{386}\ (\pm 1360) = \pm 176$ lbs.

9. $P_{30} = 176 + 50 = 226$ lbs.
$P_{90} = -176 + 50 = -126$ lbs.

10. P_n (at Pt. 30) $= \dfrac{226}{\cos 18.7°} = 240$ lbs.

P_n (at Pt. 90) $= \dfrac{-126}{\cos 13.95°} = -130$ lbs.

11. At points 30 and 90, the radius of curvature (R_c) and the normal force (P_n) are opposite in sign. Therefore, the profile is concave. For GM Meehanite and steel, the material factor (M) is 0.219 from Table H-1.

$$C_{30} = \frac{11.8}{(11.8 + 0.75)(0.75)} = 1.25$$

$$S_c = (10)^3\sqrt{\frac{240\ (1.25)}{1.0\ (0.219)}} = 37{,}000 \text{ psi.}$$

$$C_{90} = \frac{2.1}{(2.1 + 0.75)(0.75)} = 0.98$$

$$S_c = (10)^3\sqrt{\frac{130\ (0.98)}{1.0\ (0.219)}} = 24{,}000 \text{ psi.}$$

It is recommended that the results of the calculations be tabulated as shown in Table J-1.

Point	y	R_n	γ	R_c	a	$\frac{W}{g}a$	P	P_n	S_c
0	0	3.250	0			0	50	50	
30	0.136	3.386	18.70	-11.8	1360	176	226	240	37000
31	0.149	3.399	19.50						
60	0.750	4.000	29.86			0	50		
90	1.363	4.613	13.95	2.1	-1360	-176	-126	-130	24000
91	1.376	4.626	13.21						
120	1.500	4.75	0			0	50	50	

TABLE J-1

Cam Curve Synthesis

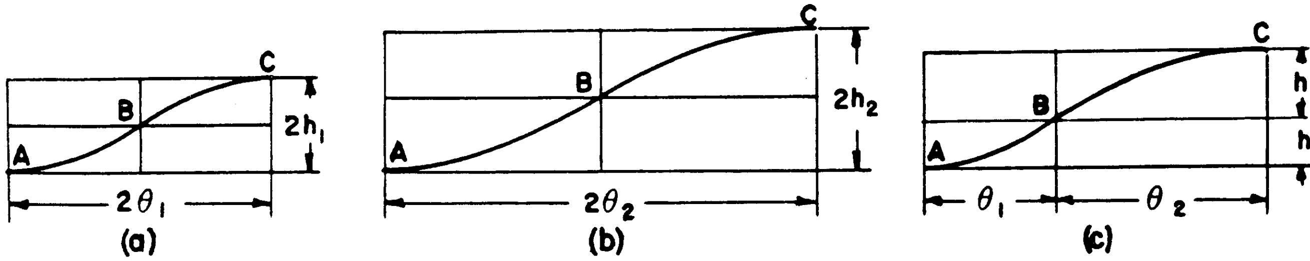


FIG. K-1

Definition. Cam curve synthesis is the process of coupling segments of the basic curves in a manner that will provide continuous displacement, velocity, and acceleration curves.

Curve Selection. The cycloidal curve, having zero acceleration at its terminals, is particularly suited to the dwell-rise-dwell condition. It is not recommended when the return follows the rise with no intermediate dwell, as the unnecessary return to zero acceleration is objectionable. In combination with the harmonic, constant acceleration and constant velocity curves, the cycloidal can provide effective solution to the majority of functional situations encountered in cam design.

The constant acceleration or constant velocity curves should not be used by themselves, except at very low speeds.

General Synthesis Procedure. It will be noted that all the basic curves, except the constant acceleration, have zero acceleration at the mid-point. Therefore, if two different curves are coupled at this point, a continuous acceleration curve is assured. If the displacement curve is to be smooth, the pressure angles at the coupling points must be identical for both curves. But the pressure angle is a function of the velocity; so, if the velocities match, the displacement curve will be smooth and continuous, and the velocity curve, at least, continuous. Therefore, if two curves are coupled at their mid-points, it is only necessary to have matching velocities at the juncture. To meet this condition, certain relations between the linear and angular displacements must be maintained. The following shows the general procedure.

Segment AB of Fig. K-1(a) is to be coupled to segment BC of Fig. K-1(b), to form the curve shown in Fig. K-1(c).

The general equation for the velocity from Section E, Equation 2, is

$$v = C_v h\left(\frac{6N}{\beta}\right)$$

Let the coefficient of velocity (C_v) at B of Fig. K-1(a) $= C_1$, and at B of Fig. K-1(b) $= C_2$; also $h = 2h_1$ and $2h_2$ and $\beta = 2\theta_1$ and $2\theta_2$ for the respective curves. Substituting these values in the equation and equating the velocities,

$$v = C_1(2h_1)\left(\frac{6N}{2\theta_1}\right) = C_2(2h_2)\left(\frac{6N}{2\theta_2}\right)$$

From this

$$\frac{C_1 h_1}{C_2 h_2} = \frac{\theta_1}{\theta_2} \tag{1}$$

This equation shows the necessary relations between linear and angular displacements for correct curve synthesis. The following examples are typical of the most usual conditions encountered.

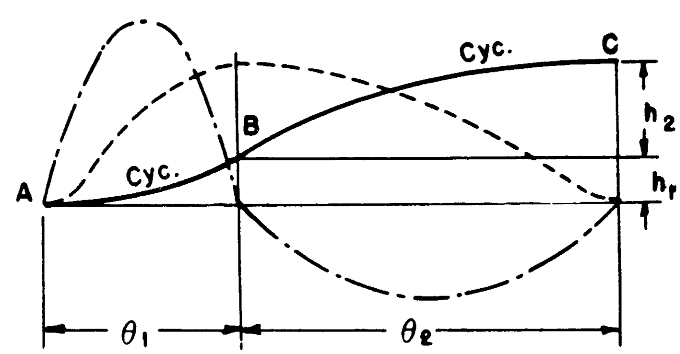


FIG. K-2

Cycloidal to Cycloidal. See Fig. K-2. From Table E-4, Section E, the coefficient of velocity (C_v) = 2.0 at the mid-points. Substituting in Equation 1,

$$\frac{2h_1}{2h_2} = \frac{\theta_1}{\theta_2} \quad \text{or} \quad \frac{h_1}{h_2} = \frac{\theta_1}{\theta_2} \tag{2}$$

This equation shows that the linear and angular displacements of the two half-cycloidal curves are directly proportional.

It is sometimes desirable to control the relation between the maximum positive and negative accelerations. The following equation establishes this control.

$$\theta_1 = n\theta_2 \tag{3}$$

where n is the desired ratio.

EXAMPLE No. 1. In Fig. K-2, let $h_1 = 0.25$; $h_2 = 0.375$; $\theta_1 = 40°$. Find θ_2. From Equation 2,

$$\frac{0.25}{0.375} = \frac{40}{\theta_2}$$

Therefore

$$0.25\theta_2 = 40\ (0.375) = 15$$

and

$$\theta_2 = \frac{15}{0.25} = 60°$$

EXAMPLE No. 2. A cam is to have a cycloidal rise of 1.2 in. in 90° between dwells. The peak negative acceleration is to be numerically one-half the peak positive acceleration. From Equation 3:

$$\theta_1 = 0.5\theta_2$$

but

$$\theta_1 + \theta_2 = 90°$$

Therefore,

$$\begin{aligned} 1.5\theta_2 &= 90° \\ \theta_2 &= 60° \\ \theta_1 &= 90 - 60 = 30° \end{aligned}$$

From Equation 2:

$$\frac{h_1}{h_2} = \frac{30}{60} \quad \text{or} \quad h_2 = 2h_1$$

but

$$h_1 + h_2 = 1.2$$

Therefore

$$3h_1 = 1.2$$

and

$$\begin{aligned} h_1 &= 0.4 \text{ in.} \\ h_2 &= 1.2 - 0.4 = 0.8 \text{ in.} \end{aligned}$$

The velocity and acceleration curves for this example are shown in Fig. K-2.

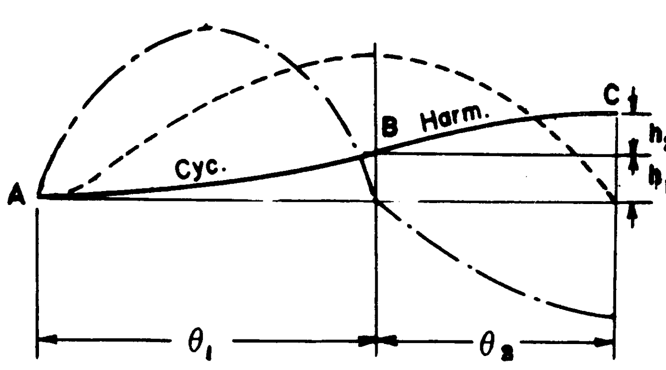


FIG. K-3

Cycloidal to Harmonic. This combination is useful for a dwell-rise-return cam. Consider the schematic displacement diagram Fig. K-3, in which the segment AB is cycloidal and the segment BC, harmonic. The coefficient of velocity at point B is 2 for the cycloidal; 1.5708 for the harmonic, from Tables E-4 and E-3 respectively. Substituting in Equation 1,

$$\frac{2h_1}{1.5708h_2} = \frac{\theta_1}{\theta_2} \tag{4}$$

This equation represents the ratio of linear and angular displacements for this combination. Further discussion of these conditions will be presented in Section L.

EXAMPLE No. 3. A symmetrical dwell-rise-return cam has a rise of 0.5 in. in 30° of a 50° total angular displacement. Find the total linear displacement required. In Fig. K-3, $h_1 = 0.5$; $\theta_1 = 30°$; $\theta_2 = 20°$. Substituting in Equation 4,

$$\frac{2\ (0.5)}{1.5708h_2} = \frac{30}{20}$$

Solving

$$h_2 = 0.424$$

Therefore

$$h_1 + h_2 = 0.5 + 0.424 = 0.924$$

The velocity and acceleration diagrams for this example are shown in Fig. K-3.

Cycloidal to Constant Velocity. This combination is used when the design requires a fixed velocity at some section of the cam. It is also valuable when fixed cam dimensions result in a greater pressure angle than desired if a full cycloidal curve is used. The maximum pressure angle will be reduced but the peak accelerations will be greater than those of the full cycloidal curve.

In the displacement diagram, Fig. K-4, the segment AB is cycloidal, the segment BC is constant velocity.

The corresponding coefficients of velocity at point B from Tables E-4 and E-1 are 2.0 and 1.0. Substituting in Equation 1,

$$\frac{2h_1}{h_2} = \frac{\theta_1}{\theta_2} \tag{5}$$

This is the ratio of linear and angular displacements for the combined curves.

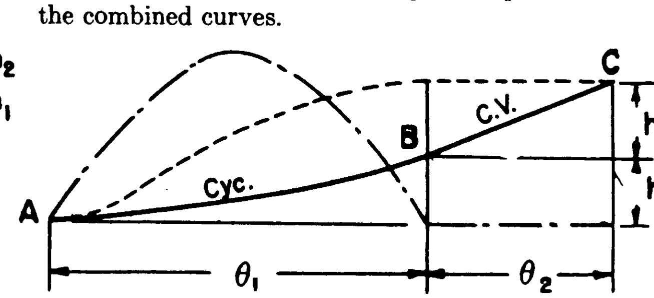


FIG. K-4

EXAMPLE NO. 4. Let $h_1 = 0.75$; $h_2 = 0.75$; $\theta_2 = 50°$. Find θ_1. From Equation 5,

$$\frac{2\ (0.75)}{0.75} = \frac{\theta_1}{50}$$

Solving,

$$\theta_1 = 100°$$

The velocity and acceleration diagrams are shown in Fig. K-4.

EXAMPLE NO. 5. As shown in Fig. K-5, a cam is to have a 0.25 in. cycloidal rise coupled to a constant velocity curve which acts through 30°, and is in turn coupled to a cycloidal curve. The event begins and ends in a dwell, the total displacement of 1 in. occurring in 100°. From Equation 5,

$$\frac{2\ (0.25)}{h_2} = \frac{\theta_1}{30}$$

and

$$h_2 = \frac{15}{\theta_1} \tag{6}$$

From Equation 2,

$$\frac{0.25}{h_3} = \frac{\theta_1}{\theta_3} \tag{7}$$

However,

$$h_3 = 1.00 - 0.25 - h_2 = 0.75 - \frac{15}{\theta_1} \tag{8}$$

$$\theta_3 = 100 - 30 - \theta_1 = 70 - \theta_1 \tag{9}$$

Substituting in Equation 7,

$$\frac{0.25}{0.75 - (15/\theta_1)} = \frac{\theta_1}{70 - \theta_1}$$

Solving,

$$\theta_1 = 32.5°$$

From Equation 9

$$\theta_3 = 70 - 32.5 = 37.5°$$

From Equation 6,

$$h_2 = \frac{15}{32.5} = 0.4615 \text{ in.}$$

From Equation 8,

$$h_3 = 0.75 - 0.4615 = 0.2885$$

Use of Tables for Half Curves. To use the displacement, velocity, and acceleration tables, adjustments must be made for half curves.

For the first half of a basic curve, points 0 to 60, Fig. K-6(a) the equations are

$$y = 2Kh_1 \tag{10}$$

$$v = 2C_v h_1\left(\frac{6N}{2\theta_1}\right) \tag{11}$$

$$a = 2C_a h_1\left(\frac{6N}{2\theta_1}\right)^2 \tag{12}$$

For the second half, points 60 to 120, Fig. K-6(b)

$$y = (2K - 1)h_2 \tag{13}$$

$$v = 2C_v h_2\left(\frac{6N}{2\theta_2}\right) \tag{14}$$

$$a = 2C_a h_2\left(\frac{6N}{2\theta_2}\right)^2 \tag{15}$$

Pressure Angles. Pressure angles may be calculated by

$$\tan\gamma = \frac{57.3C_v h_1}{R_n\theta_1} \tag{16}$$

$$\tan\gamma = \frac{57.3C_v h_2}{R_n\theta_2} \tag{17}$$

$$\tan\gamma = \frac{9.55v}{R_n N} \tag{18}$$

Reference

1. Plate Cam Design. M. Kloomok and R. V. Muffley. Prod. Eng. Feb. 1955.

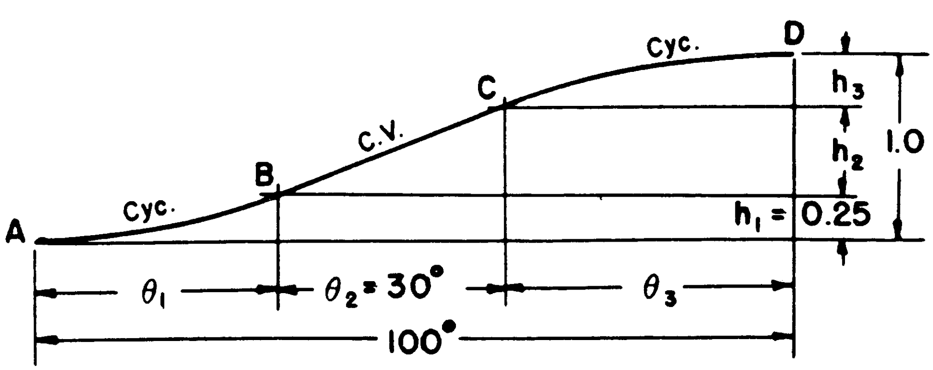


FIG. K-5

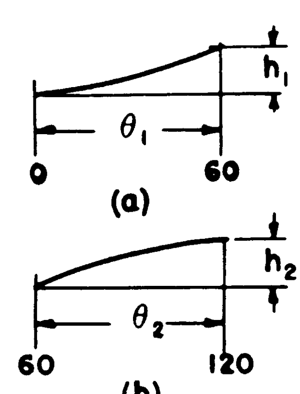


FIG. K-6

Dwell-Rise-Return-Dwell Cam Using Basic Curves

Symmetrical Rise and Return. In Fig. L-1(a) is shown the displacement diagram of a typical cam of this operational sequence. Note that the rise and return take place through the same cam angle. In Fig. L-1(b) are shown cycloidal, harmonic, and constant acceleration diagrams of the same conditions. Note that the cycloidal curve has an abrupt change of acceleration at the maximum rise point. This is objectionable as it can induce undue vibration. This characteristic is not present in the harmonic and constant acceleration curves, but there is instantaneous acceleration at the terminals. Therefore none of these curves is quite satisfactory for the D-R-R-D cam.

A combination of the cycloidal and the harmonic curves retains the good characteristics of both. The cycloidal provides zero acceleration at the terminals and the harmonic, continuous acceleration at the maximum rise point.

A typical diagram of this combination appears in Fig. L-2.

Equation Development. In Fig. L-2, let the relation between θ_1 and θ_2 be n; thus, $\theta_2 = n\theta_1$. From Section K, Equation 4

$$\frac{2h_1}{1.5708h_2} = \frac{\theta_1}{n\theta_1} = \frac{1}{n}$$

Therefore

$$2h_1 n = 1.5708h_2 = 1.5708(h - h_1) \tag{1}$$

and

$$h_1 = \frac{\pi}{\pi + 4n}\, h \tag{2}$$

Also

$$h_2 = \frac{4n}{\pi + 4n}\, h \tag{3}$$

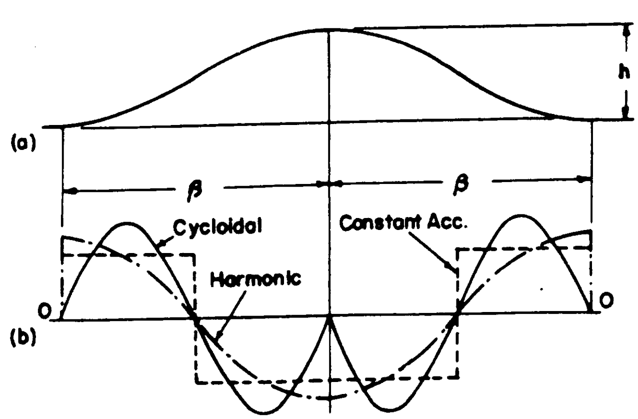


FIG. L-1

The following equations can be derived for the velocity and acceleration:

The maximum velocity at B is

$$v_B = \frac{\pi(n+1)^2 h}{\pi + 4n}\left(\frac{6N}{\beta}\right) \tag{4}$$

The maximum positive acceleration at M is

$$a_M = \frac{\pi^2(n+1)^2 h}{\pi + 4n}\left(\frac{6N}{\beta}\right)^2 \tag{5}$$

The maximum negative acceleration at C is

$$a_C = -\frac{\pi^2(n+1)^2 h}{n(\pi + 4n)}\left(\frac{6N}{\beta}\right)^2 \tag{6}$$

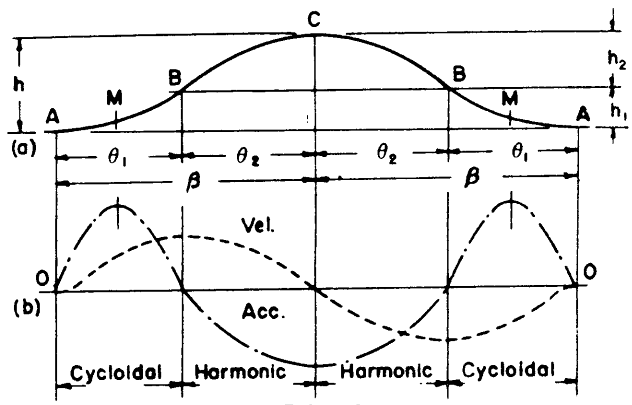


FIG. L-2

If $\theta_1 = \theta_2$, then $n = 1.0$ and the above equations become

$$h_1 = 0.4399h \tag{7}$$

$$h_2 = 0.5601h \tag{8}$$

$$v_B = 1.7596h\left(\frac{6N}{\beta}\right) \tag{9}$$

$$a_M = 5.528h\left(\frac{6N}{\beta}\right)^2 \tag{10}$$

$$a_C = -5.528h\left(\frac{6N}{\beta}\right)^2 \tag{11}$$

Unsymmetrical Rise and Return. If the rise and return occur through different cam angles, as shown in Fig. L-3(a), a difficulty arises at the maximum rise point. Unless the cycloidal and harmonic divisions of angles β_1 and β_2 are properly adjusted, there will be a break in the continuity of the acceleration curve as shown in Fig. L-3(b). This discontinuity should be avoided, or, at least, minimized.

The following procedure gives a satisfactory solution when the ratio between the rise and return angles does not exceed 1.5.

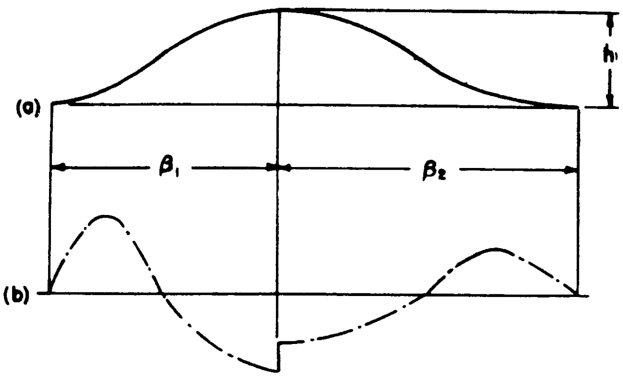


FIG. L-3

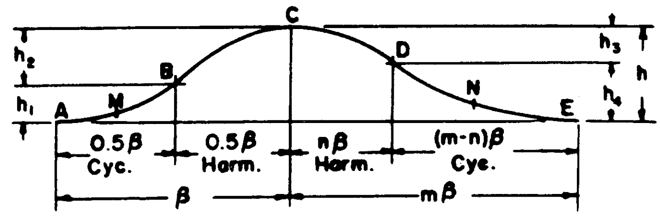


FIG. L-4

Consider the displacement diagram, Fig. L-4, which shows the pertinent symbols. The lesser angle β is divided into two equal periods. Curve AB is a half-cycloidal curve; curve BC is half-harmonic. The characteristics have already been determined in Equations 7 through 11.

The greater cam angle is designated $m\beta$, where m is the ratio of the greater to the lesser angle. This curve is also divided into a half-harmonic section CD and a cycloidal section DE. The relation n between the CD curve angle ($n\beta$) and β is to be determined. For a continuous acceleration curve, the acceleration at C should be the same for both rise and return.

For curve CD, from Section K, Equation 15 and Table E-3

$$a_C = -4.9348\,(2h_3)\left(\frac{6N}{2n\beta}\right)^2 \qquad (12)$$

Combining Equations 11 and 12, and solving for h_3

$$h_3 = 2.2404n^2h \qquad (13)$$

Also

$$h_4 = (1 - 2.2404n^2)h \qquad (14)$$

The velocity at D should be the same for curves CD and DE.

From Section K, Equation 14 and Table E-3, for curve CD

$$v_D = 2\,(1.5708)(2.2404n^2h)\left(\frac{6N}{2n\beta}\right) \qquad (15)$$

From Section K, Equation 11 and Table E-4, for curve DE

$$v_D = 2\,(2h)(1 - 2.2404n^2)\left[\frac{6N}{2(m - n)\beta}\right] \qquad (16)$$

Combining Equations 15 and 16 results in the quadratic equation

$$0.9616n^2 + 3.5192mn - 2 = 0 \qquad (17)$$

Solving for n

$$n = \frac{\sqrt{(3.5192m)^2 + 7.6928} - 3.5192m}{1.9232} \qquad (18)$$

Having n, all other characteristics of curve CDE can be derived.

From Equations 15 or 16, the maximum velocity is

$$v_D = 3.5192hn\left(\frac{6N}{\beta}\right) \qquad (19)$$

From Section K, Equation 15, the maximum positive acceleration is

$$a_N = \frac{\pi h\,(1 - 2.2404n^2)}{(m - n)^2}\left(\frac{6N}{\beta}\right)^2 \qquad (20)$$

For convenience of use, the characteristic equations are summarized below.

Curve ABC

$$h_1 = 0.4399h \qquad (7)$$

$$h_2 = 0.5601h \qquad (8)$$

$$v_B\ (\text{max.}) = 1.7596h\left(\frac{6N}{\beta}\right) \qquad (9)$$

$$a_M\ (\text{max. pos.}) = 5.528h\left(\frac{6N}{\beta}\right)^2 \qquad (10)$$

$$a_C\ (\text{max. neg.}) = -5.528h\left(\frac{6N}{\beta}\right)^2 \qquad (11)$$

Curve CDE

$$h_3 = 2.2404n^2h \qquad (13)$$

$$h_4 = (1 - 2.2404n^2)h \qquad (14)$$

$$v_D\ (\text{max.}) = 3.5192hn\left(\frac{6N}{\beta}\right) \qquad (19)$$

$$a_C\ (\text{max. neg.}) = -5.528h\left(\frac{6N}{\beta}\right)^2 \qquad (11)$$

$$a_N\ (\text{max. pos.}) = \frac{\pi h\,(1 - 2.2404n^2)}{(m - n)^2}\left(\frac{6N}{\beta}\right)^2 \qquad (20)$$

Design Example. In Fig. L-4 let $h = 0.5$ in.; $\beta = 40°$; $m\beta = 50°$; $N = 100$ RPM; $m = 50/40 = 1.25$.

From Equation 7,

$$h_1 = 0.4399\,(0.5) = 0.220 \text{ in.}$$

From Equation 8,

$$h_2 = 0.5601\ (0.5) = 0.280 \text{ in.}$$

From Equation 18,

$$n = \frac{\sqrt{(3.5192 \times 1.25)^2 + 7.6928} - (3.5192 \times 1.25)}{1.9232} = 0.4167$$

Therefore

$$n\beta = 0.4167\ (40) = 16.668°$$
$$(m - n)\beta = (1.25 - 0.4167)\ (40) = 33.332°$$

From Equation 13,

$$h_3 = 2.2404\ (0.4167)^2\ (0.5) = 0.195 \text{ in.}$$

From Equation 14,

$$h_4 = [1 - 2.2404\ (0.4167)^2]\ (0.5) = 0.305 \text{ in.}$$

From Equation 11,

$$a_C = -5.528\ (0.5)\left(\frac{6 \times 100}{40}\right)^2 = -620 \text{ in./sec.}^2$$

It may be desirable to have $n\beta$ in integral degrees. This can usually be done with a slight discrepancy in accelerations.

If $n\beta$ is made 17° instead of 16.686°; $n = 17/40 = 0.425$.

From Equation 13,

$$h_3 = 2.2404\ (0.425)^2\ (0.5) = 0.203 \text{ in.}$$
$$h_4 = 0.500 - 0.203 = 0.297 \text{ in.}$$

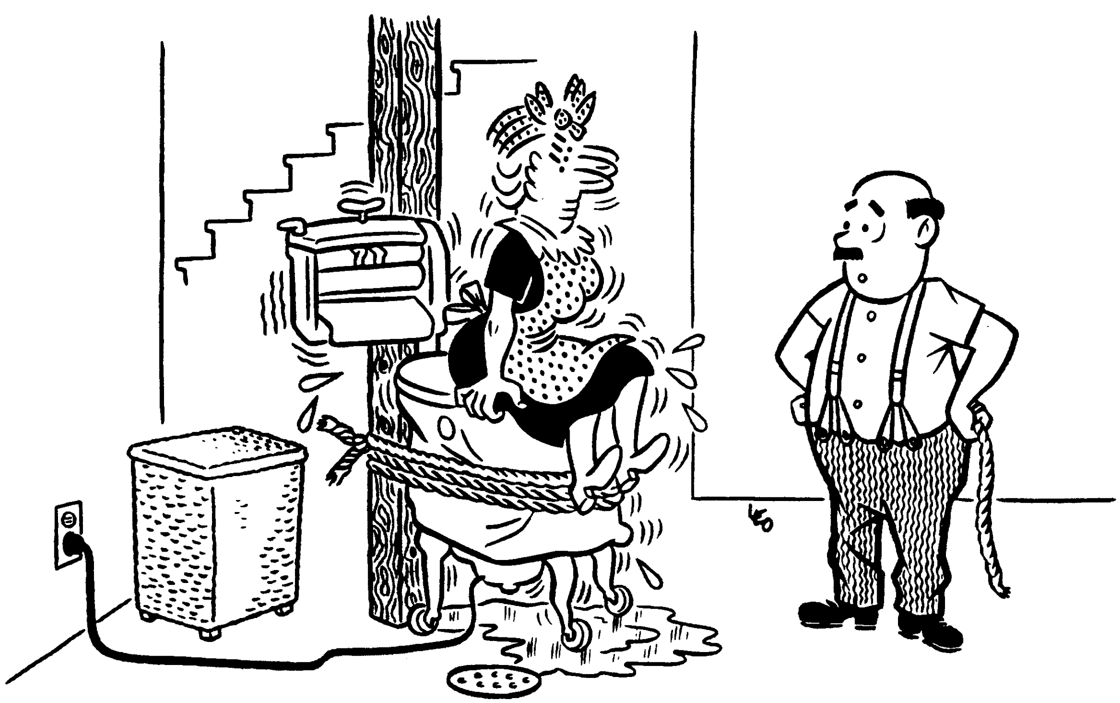
From Equation 12,

$$a_C = -4.9348\ (2)(0.203)\left(\frac{6 \times 100}{2 \times 17}\right)^2 = -635 \text{ in./sec.}^2$$

The difference in acceleration is negligible.

Reference

1. Continuous Cam Curves. Edgar Schmidt. Machine Design. Jan. 7, 1960.



UNSYMMETRICAL RISE & FALL

Cam Curve with Terminal Velocity

Operational Conditions. Sequence of operational functions of a cam sometimes necessitate coupling a terminal of a basic curve to another curve which has a fixed terminal velocity. To assure continuous velocity and acceleration, it is then necessary to adjust the basic curve to the required terminal velocity. The method is shown by the following procedure.

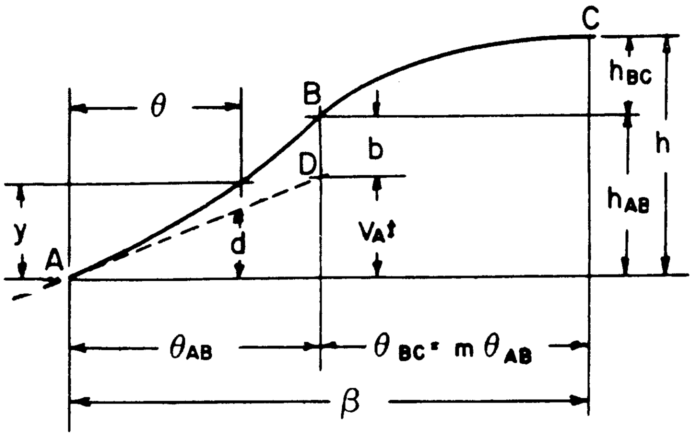


FIG. M-1

Equation Development. Consider the displacement diagram in Fig. M-1. The terminal point A has a fixed velocity, (v_A); the velocity at point C, (v_C) is zero; the curve BC, ending in a dwell, is a half-cycloid; the curve AB, a half-cycloid adjusted to the terminal velocity v_A.

To develop the equations, note that the displacement at any point of the constant velocity line extended through A to D is

$$d = v_A t = v_A \frac{\theta}{6N} \tag{1}$$

where θ = angular displacement (deg.) at any point; t = time (sec.) for the cam to revolve through θ; N = RPM of cam shaft.

Also, note that the displacement of the adjusted half-cycloid at point B is

$$b = h_{AB} - v_A t = h_{AB} - v_A \frac{\theta_{AB}}{6N} \tag{2}$$

and that the displacement of the half-cycloid BC from point B is

$$h_{BC} = h - h_{AB} \tag{3}$$

Also, the acceleration at B for both curves is zero.

At point B, the velocity (v_B) must be the same for curves AB and BC. For curve AB, v_B is the velocity as determined from Section K; Equation 11 plus the terminal velocity v_A.

$$v_B = 2\,(2)\left(h_{AB} - v_A \frac{\theta_{AB}}{6N}\right)\frac{6N}{2\theta_{AB}} + v_A \tag{4}$$

For curve BC, from Section K; Equation 14

$$v_B = 2(2)\,(h - h_{AB})\,\frac{6N}{2m\theta_{AB}} \tag{5}$$

Combining Equations 4 and 5, and solving for h_{AB}

$$h_{AB} = \frac{1}{m+1}\left(h + 0.5v_A \frac{m\theta_{AB}}{6N}\right) \tag{6}$$

Also

$$h_{BC} = \frac{m}{m+1}\left(h - 0.5v_A \frac{m\theta_{AB}}{6N}\right) \tag{7}$$

If $\theta_{AB} = \theta_{BC}$, $m = 1$ and

$$h_{AB} = 0.5\left(h + 0.5v_A \frac{\theta_{AB}}{6N}\right) \tag{8}$$

$$h_{BC} = 0.5\left(h - 0.5v_A \frac{\theta_{AB}}{6N}\right) \tag{9}$$

Other characteristics may be determined from the following.

For curve AB:

$$\text{Displacement:} \quad y = 2Kb + v_A \frac{\theta}{6N} \tag{10}$$

where K = displacement factor; θ = angular displacement of reference point,

$$\text{Velocity:} \quad v = C_v h_{AB} \frac{6N}{\theta_{AB}} - v_A(C_v - 1.0) \tag{11}$$

$$\text{Acceleration:} \quad a = 0.5C_a\left[h_{AB}\left(\frac{6N}{\theta_{AB}}\right)^2 - v_A \frac{6N}{\theta_{AB}}\right] \tag{12}$$

For curve BC:

$$\text{Displacement:} \quad y = h_{AB} + 2Kh_{BC} \tag{13}$$

$$\text{Velocity:} \quad v = C_v h_{BC} \frac{6N}{\theta_{BC}} \tag{14}$$

$$\text{Acceleration:} \quad a = 0.5C_a h_{BC}\left(\frac{6N}{\theta_{BC}}\right)^2 \tag{15}$$

Design Example No. 1. Let h = 1.5 in., $\theta_{AB} = 30°$; $\theta_{BC} = 24°$; v_A = 6 in./sec.; N = 100 RPM. Determine the displacements and peak velocity and accelerations.

$$m = 24/30 = 0.8$$

From Equation 6,

$$h_{AB} = \frac{1}{1.8}\left(1.5 + 3\,\frac{24}{600}\right) = 0.9 \text{ in.}$$

From Equation 7,

$$h_{BC} = \frac{0.8}{1.8}\left(1.5 - 3\,\frac{24}{600}\right) = 0.6 \text{ in.}$$

From Equation 11,

$$v_B = 2\,(0.9)\left(\frac{600}{30}\right) - 6\,(2 - 1) = 30 \text{ in./sec.}$$

From Equation 14,

$$v_B = 2\,(0.6)\left(\frac{600}{24}\right) = 30 \text{ in./sec.} \qquad \text{(Check)}$$

From Equation 12,

$$a\ (\text{max. pos.}) = 0.5\,(6.2832) \times \left[0.9\left(\frac{600}{30}\right)^2 - 6\left(\frac{600}{30}\right)\right] = 754 \text{ in./sec.}^2$$

From Equation 15,

$$a\ (\text{max. neg.}) = 0.5\,(-6.2832)(0.6)\left(\frac{600}{24}\right)^2 = -1178 \text{ in./sec.}^2$$

Design Example No. 2. A cam operated tool is applied to a work piece held in a receptacle mounted on a six-station geneva driven dial. The tool makes contact with the work $^3/_8$ in. from the end of the stroke at a relatively low velocity, continues at constant velocity for a time, and then decelerates to zero and dwells under pressure for a certain period. The tool is then withdrawn and the cycle repeated. Design conditions require at least $^3/_4$ in. retraction of the tool before the start of the indexing, and not more than $^1/_4$ in. advance before the dial stops. The total displacement of the tool is $1^1/_4$ in.

This example not only presents a problem of curve synthesis, but also a condition where intermediate displacements must be aligned with angular displacements.

The displacement diagram Fig. M-2 shows the known and desired conditions. Note curve AB is a full cycloid; CD, a half-cycloid; DE, a velocity-adjusted cycloid; EF, constant velocity; and FG, a half cycloid.

The procedure is as follows:

1. Determine h_{EF}, h_{FG}, θ_{EF}, θ_{FG}; from Section K; Equation 5

$$\frac{2h_{FG}}{h_{EF}} = \frac{\theta_{FG}}{\theta_{EF}} \qquad (16)$$

Knowing that $h_{EF} + h_{FG} = 0.375$, and $\theta_{EF} + \theta_{FG} = 100°$, if one value is assumed, the others may be determined.

Let $\theta_{EF} = 50°$ and substitute in Equation 16

$$\frac{2\,(0.375 - h_{EF})}{h_{EF}} = \frac{100 - 50}{50}$$

Solving

$$h_{EF} = 0.25 \text{ in.} \qquad (17)$$

and

$$h_{FG} = 0.125 \text{ in.} \qquad (18)$$

$$\theta_{EF} = 50° \qquad (19)$$

$$\theta_{FG} = 50° \qquad (20)$$

2. Determine the velocity at E or F at one RPM.

From Section K; Equation 11 and Tables E-1 and E-4

$$v_E = 2\,(1.0)(0.25)\,\frac{6}{2\,(50)} = 0.03 \text{ in./sec.} \qquad (21)$$

$$v_E = 2\,(2.0)(0.125)\,\frac{6}{2\,(50)} = 0.03 \text{ in./sec.} \qquad (22)$$

3. Determine θ_{AP} and θ_{QE}.

There is available, $360° - (120 + 100 + 60) = 80°$

Making the angular displacements proportional to the linear displacements

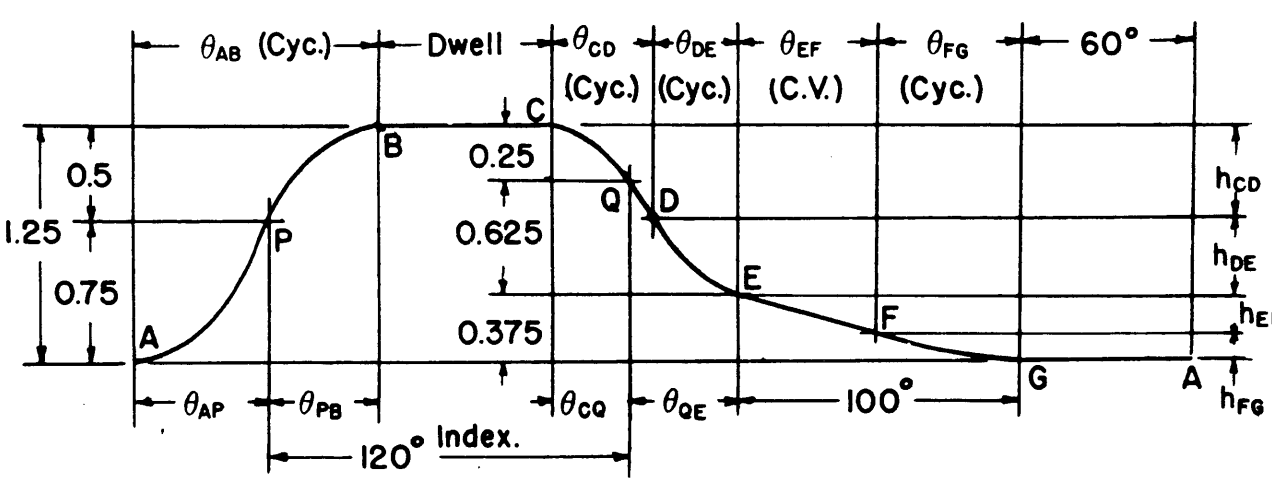


FIG. M-2

$$\frac{\theta_{AP}}{\theta_{AP} + \theta_{QE}} = \frac{h_{AP}}{h_{AP} + h_{QE}}$$

or

$$\frac{\theta_{AP}}{80} = \frac{0.75}{1.375}$$

Solving

$$\theta_{AP} = 44° \tag{23}$$

and

$$\theta_{QE} = 36° \tag{24}$$

4. Determine θ_{AB}.

Note that $h_{AP} = 0.75$ which is 0.75/1.25 or 0.6 h_{AB}.

From Table E-4, the nearest value to 0.6 is 0.59918 occurring at the 66th point.

Therefore

$$\theta_{AB} = 120\ (44/66) = 80° \tag{25}$$

and

$$h_{AP} = 0.59918\ (1.25) = 0.749\ \text{(Check)}$$

5. Determine θ_{CE}.

Again making angular displacements proportional to the linear displacements,

$$\frac{\theta_{CE}}{\theta_{AB}} = \frac{h_{CE}}{h_{AB}} \quad \text{or} \quad \frac{\theta_{CE}}{80} = \frac{0.875}{1.25}$$

and

$$\theta_{CE} = 56° \tag{26}$$

6. Determine h_{CD} and h_{DE}.

Although θ_{CE} may be divided arbitrarily, division into equal parts is usually a satisfactory choice. Therefore,

$$\theta_{CD} = \theta_{DE} = 56/2 = 28° \tag{27}$$

From Equation 8 for $m = 1$

$$h_{DE} = 0.5\left(h_{CE} + 0.5 r_E \frac{\theta_{DE}}{6}\right)$$

Substituting the known values

$$h_{DE} = 0.5\left[0.875 + 0.5\ (0.03)\ \frac{28}{6}\right] = 0.4725\ \text{in.} \tag{28}$$

From Equation 9

$$h_{CD} = 0.4025\ \text{in.} \tag{29}$$

7. Check h_{CQ} for conformity to specifications.

$$\theta_{CQ} = \frac{20}{2\ (28)} = \frac{20}{56} \text{ of a full cycloidal period}$$

$$\frac{20}{56}\ (120) = 43$$

The displacement factor corresponding to the 43rd point of a full curve is 0.23465 from Table E-4. Therefore, from Section K; Equation 10,

$$h_{CQ} = 2\ (0.23465)(0.4025) = 0.189\ \text{in.} \tag{30}$$

which is well within the specified 0.25 in.

The final displacement diagram is shown in Fig. M-3.

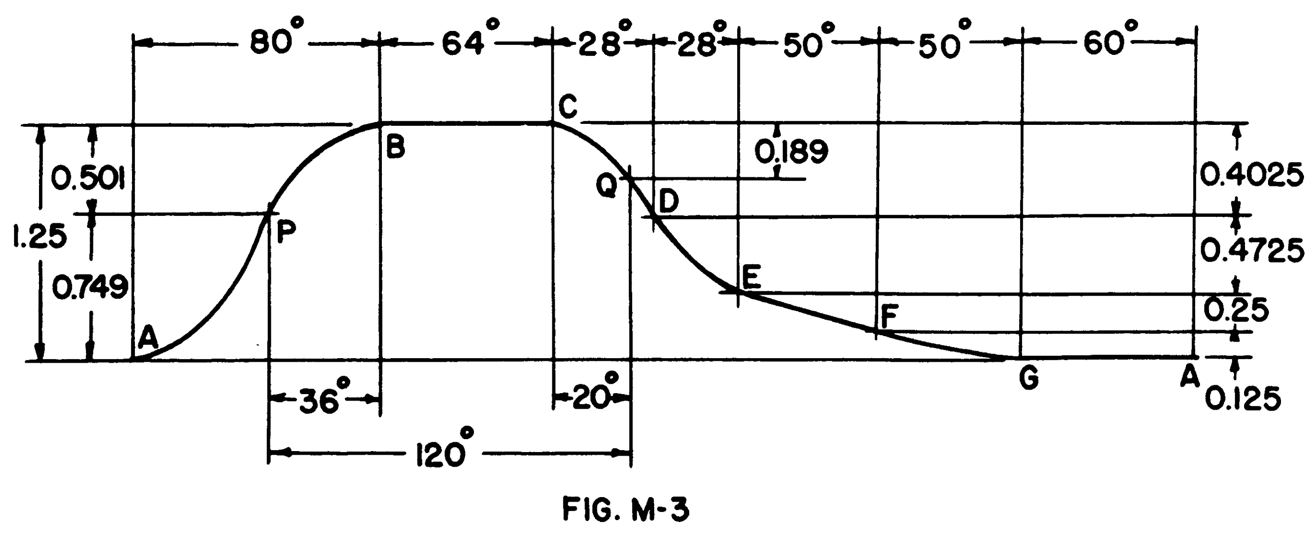


FIG. M-3

SECTION N

Modified Trapezoid and Modified Sine Curves

Modified Trapezoid. This curve is a combination of the cycloidal and constant acceleration curves. It has lower peak acceleration than the cycloidal, while retaining the advantages of zero acceleration at the terminals. It has one objectionable characteristic: the torque goes from maximum positive to maximum negative in one-fifth of the travel time. If dynamic forces represent the major part of the load on the cam, the comparatively sudden release of energy may be detrimental. Much better torque characteristics are displayed by the cycloidal and modified sine curve.

Development of Curve. In Fig. N-1 are shown the displacement and acceleration diagrams of the modified trapezoid curve; and in Fig. N-2 the basic cycloidal curves from which it is developed. Pertinent symbols and relationship of angular displacements are indicated in these figures. Note that primed symbols refer to the basic curve.

From Table E-4, for cycloidal displacement, at point 30, corresponding to point B′ of the basic curve

$$h'_{A'B'} = 0.09085h' \tag{1}$$

Let $h'_{A'B'} = h_{AB}$, then

$$h_{AB} = 0.09085h' \tag{2}$$

Let the velocity at B′ equal the velocity at B. As only relative values are involved, let the RPM = 1.0. Then from Section E; Equation 2

$$v_B = C_v h' \left(\frac{6N}{\beta'}\right) = 1.0h' \frac{6}{0.5\beta} = \frac{12h'}{\beta} \tag{3}$$

Let the acceleration at B′ equal the acceleration at B. From Section E; Equation 3

$$a_B = C_a h' \left(\frac{6N}{\beta'}\right)^2 = 6.2832h' \left(\frac{6}{0.5\beta}\right)^2 = 6.2832h' \left(\frac{12}{\beta}\right)^2 \tag{4}$$

From Section D; Equation 6 for constant acceleration

$$h_{BC} = v_B t + 0.5a_B t^2 \tag{5}$$

But

$$t = \frac{0.25\beta}{6N} = \frac{\beta}{24} \tag{6}$$

Substituting Equations 3, 4, and 6 in Equation 5,

$$h_{BC} = 1.2854h' \tag{7}$$

To determine the displacement at D, for the constant acceleration, from Section D; Equation 4

$$v_C = v_B + a_B t$$

Substituting Equations 3 and 4

$$v_C = 4.1416h' \left(\frac{12}{\beta}\right) \tag{8}$$

From Fig. D-7, the construction of the cycloidal curve, it may be seen that the displacements are the sum of a constant velocity displacement and a harmonic displacement. Therefore

$$h_{CD} = v_C t + \frac{h'}{2\pi}$$

Noting that t for period between C and D is $0.125\beta/6$, and substituting Equation 8

$$h_{CD} = 1.19455h' \tag{9}$$

It is apparent that

$$h_{AB} + h_{BC} + h_{CD} = 0.5h$$

Substituting Equations 2, 7, and 9

$$0.09085h' + 1.2854h' + 1.19455h' = 0.5h$$

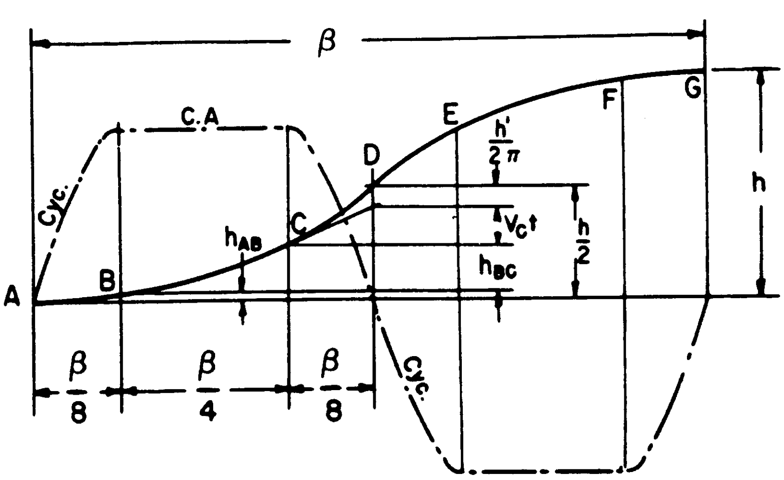


FIG. N-1
MODIFIED TRAPEZOID CURVE

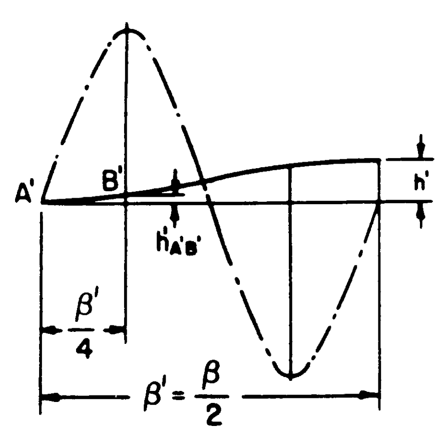


FIG. N-2

From this

$$h' = 0.1945h \tag{10}$$

From Equation 2,

$$h_{AB} = 0.0177h \tag{11}$$

From Equation 7,

$$h_{BC} = 0.2500h \tag{12}$$

From Equation 9,

$$h_{CD} = 0.2323h \tag{13}$$

Equations 11, 12, and 13 establish the intermediate displacements in terms of the full displacement. The second half of the curve is the reverse of the first half.

Curve Characteristics. The characteristics of the modified trapezoid are as follows:

From A to B (points 0 to 15)—

$$K = 0.3890 \frac{\theta}{\beta} - 0.03095 \sin 4\pi \frac{\theta}{\beta} \tag{14}$$

$$C_v = 0.3890 - 0.3890 \cos 4\pi \frac{\theta}{\beta} \tag{15}$$

$$C_a = 4.88812 \sin 4\pi \frac{\theta}{\beta} \tag{16}$$

From B to C (points 15 to 45)—

$$K = 2.44406 \left(\frac{\theta}{\beta}\right)^2 - 0.22203 \frac{\theta}{\beta} + 0.00723 \tag{17}$$

$$C_v = 4.88812 \frac{\theta}{\beta} - 0.22203 \tag{18}$$

$$C_a = 4.88812 \tag{19}$$

From C to D (points 45 to 60)—

$$K = 1.61102 \frac{\theta}{\beta} - 0.03095 \sin \left(4\pi \frac{\theta}{\beta} - \pi\right) - 0.30551 \tag{20}$$

$$C_v = 1.61102 - 0.38898 \cos \left(4\pi \frac{\theta}{\beta} - \pi\right) \tag{21}$$

$$C_a = 4.88812 \sin \left(4\pi \frac{\theta}{\beta} - \pi\right) \tag{22}$$

From D to E (points 60 to 75)—

$$K = 1.61102 \frac{\theta}{\beta} + 0.03095 \sin \left(4\pi \frac{\theta}{\beta} - 2\pi\right) - 0.30551 \tag{23}$$

$$C_v = 1.61102 + 0.38898 \cos \left(4\pi \frac{\theta}{\beta} - 2\pi\right) \tag{24}$$

$$C_a = -4.88812 \sin \left(4\pi \frac{\theta}{\beta} - 2\pi\right) \tag{25}$$

From E to F (points 75 to 105)—

$$K = 4.66609 \frac{\theta}{\beta} - 2.44406 \left(\frac{\theta}{\beta}\right)^2 - 1.22927 \tag{26}$$

$$C_v = 4.66609 - 4.88812 \frac{\theta}{\beta} \tag{27}$$

$$C_a = -4.88812 \tag{28}$$

From F to G (points 105 to 120)—

$$K = 0.61102 + 0.38898 \frac{\theta}{\beta} + 0.03095 \sin \left(4\pi \frac{\theta}{\beta} - 3\pi\right) \tag{29}$$

$$C_v = 0.38898 + 0.38898 \cos \left(4\pi \frac{\theta}{\beta} - 3\pi\right) \tag{30}$$

$$C_a = -4.88812 \sin \left(4\pi \frac{\theta}{\beta} - 3\pi\right) \tag{31}$$

Note angles are measured in radians in the above equations.

The displacement, velocity, and acceleration coefficients are tabulated in Section E; Table 5.

Modified Sine. This curve is a combination of cycloidal and harmonic curves. The peak accelerations are greater than the modified trapezoid, but less than the cycloidal peaks. The change from positive to negative torque occurs in 0.42 of the travel time which makes it an excellent curve for indexing large mass dials or turrets.

Development of Curve. In Fig. N-3 are shown the displacement and acceleration diagrams of the modified sine curve; and, in Fig. N-4 the basic cycloidal curves from which it is developed. Primed symbols refer to the basic cycloidal curves; double primes to the harmonic section of the curve.

From Table E-4 for cycloidal displacement at point 30, corresponding to point B′ of the basic curve

$$h'_{A'B'} = 0.09085h' \tag{32}$$

Let $h'_{A'B'} = h_{AB}$, then

$$h_{AB} = 0.09085h' \tag{33}$$

Let the velocity at B′ equal the velocity at B. As only relative values are involved, let the RPM = 1.0. Then from Section E; Equation 2

$$v_B = C_v h' \left(\frac{6N}{\beta'}\right) = 1.0h' \frac{6}{0.5\beta} = \frac{12h'}{\beta} \tag{34}$$

Let the acceleration at B′ equal the acceleration at B.

The displacement at C from point B is

$$h_{BC} = v_B t + \frac{h''}{2} \tag{35}$$

But

$$t = \frac{0.375\beta}{6} \tag{36}$$

Substituting Equations 34 and 36 in Equation 35

$$h_{BC} = 0.75h' + \frac{h''}{2} \qquad (37)$$

Let the acceleration at B′ equal the acceleration at B
From Section E; Equation 3

$$a_B = C_a h'' \left(\frac{6N}{\beta''}\right)^2 = 4.9348h'' \left(\frac{6}{0.75\beta}\right)^2 \qquad (38)$$

From Section E; Equation 3

$$a_B = C_a h' \left(\frac{6N}{\beta'}\right)^2 = 6.2832h' \left(\frac{12}{\beta}\right)^2 \qquad (39)$$

Combining Equations 38 and 39

$$h'' = 2.86478h' \qquad (40)$$

Substituting Equation 40 in Equation 37

$$h_{BC} = 2.09239h' \qquad (41)$$

It is apparent that

$$h_{AB} + h_{BC} = 0.5h$$

Substituting Equations 33 and 41

$$0.09085h' + 2.09239h' = 0.5h \qquad (42)$$

From this $h' = 0.21995h$
From Equation 40,

$$h'' = 0.63011h \qquad (43)$$

From Equation 33,

$$h_{AB} = 0.01998h \qquad (44)$$

From Equation 41,

$$h_{BC} = 0.48002h \qquad (45)$$

Equations 44 and 45 establish the intermediate displacements. The second half of the curve is the reverse of the first half.

Curve Characteristics. The characteristics of the modified sine curve are:
From A to B (points 0 to 15)

$$K = 0.43990 \frac{\theta}{\beta} - 0.03501 \sin 4\pi \frac{\theta}{\beta} \qquad (46)$$

$$C_v = 0.43990 \left(1 - \cos 4\pi \frac{\theta}{\beta}\right) \qquad (47)$$

$$C_a = 5.528 \sin 4\pi \frac{\theta}{\beta} \qquad (48)$$

From B to D (points 15 to 105)

$$K = 0.43990 \frac{\theta}{\beta} - 0.31505 \cos \left(\frac{4\pi}{3}\frac{\theta}{\beta} - \frac{\pi}{6}\right) + 0.28005 \qquad (49)$$

$$C_v = 0.43990 + 1.31967 \sin \left(\frac{4\pi}{3}\frac{\theta}{\beta} - \frac{\pi}{6}\right) \qquad (50)$$

$$C_a = 5.528 \cos \left(\frac{4\pi}{3}\frac{\theta}{\beta} - \frac{\pi}{6}\right) \qquad (51)$$

From D to E (points 105 to 120)

$$K = 0.56010 + 0.43990 \frac{\theta}{\beta} - 0.03501 \sin 2\pi \left(2\frac{\theta}{\beta} - 1\right) \qquad (52)$$

$$C_v = +0.43990 \left[1 - \cos 2\pi \left(2\frac{\theta}{\beta} - 1\right)\right] \qquad (53)$$

$$C_a = +5.528 \sin 2\pi \left(2\frac{\theta}{\beta} - 1\right) \qquad (54)$$

Note angles are measured in radians in the above equations.

The displacement, velocity, and acceleration coefficients are tabulated in Section E; Table 6.

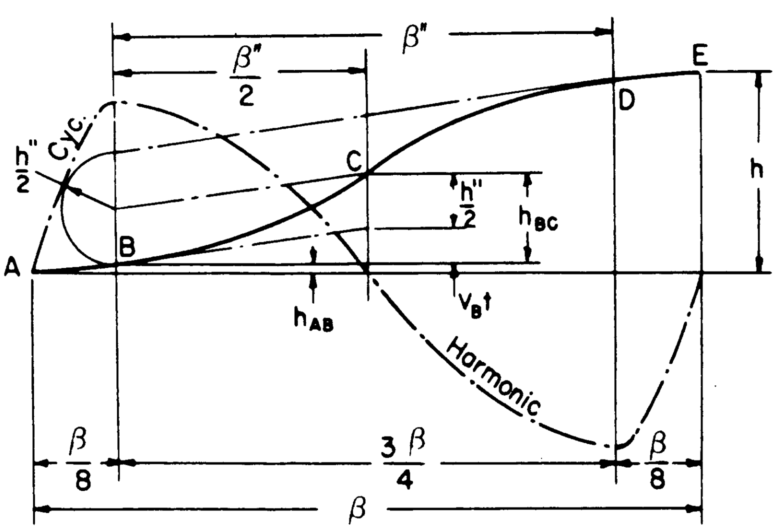


FIG. N-3
MODIFIED SINE CURVE

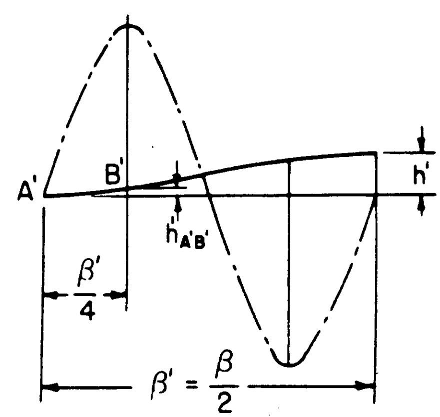


FIG. N-4

SECTION P
Cam Profile Determination

Introduction. The necessity for accurate cam profiles for moderate and high speed machinery has become increasingly evident. The method by which points on the pitch curve are determined by layout from the construction diagrams illustrated in Section D is no longer adequate. For the accuracy required point-to-point locations of the follower are calculated in polar coordinates, and a table of cam radii and corresponding cam angles is made. These tables are prepared by applying the proper displacement equations from Section D or the numerical data in Tables E-1 through E-6. Obviously it may not be necessary to use all 120 points of the table for determining the cam profile.

Cam Manufacture. It is necessary at this point to describe briefly the methods by which production cams are manufactured. Usually a master cam is prepared from the calculated data. This can be done by layout, or by increment cutting. In the layout method, the machinist constructs the profile of the cam on a cam blank, using as accurate means as available. The master cam is then machined, sawed, and filed to a scribed line. Naturally, the accuracy depends entirely on the skill of the operator.

In increment cutting, the profile is constructed by intermittent cuts, resulting in a series of circular scallops tangent to the desired profile. The machine involved is usually a jig borer. The result is a highly accurate master, although the final accuracy again depends on the skill of the workman in removing the ridges and valleys left by the scallops. This is normally a hand operation.

Whichever method of making the master cam is used, the production cams are machined on a milling machine with a cam milling attachment or on a commercial cam miller, using the master cam as the control device.

For other methods of cam manufacture, see Reference 1 at the end of the section.

Number of Division Points. In the increment cutting method the number of divisions used depends on the accuracy desired. Before the hand operation to remove the ridges the master cam will appear as shown in Fig. P-1; obviously the greater the number of divisions, the smaller the angle between data points, the smaller the scallop height, and the less material to remove.

The following equation in which r_c = radius (in.) of the cam profile from the axis; R_g = radius (in.) of the cutter or follower; and $\Delta\theta$ = angular increment (deg.) will give a scallop height (s) of approximately 0.003 in.

$$\Delta\theta = \frac{9}{r_c}\sqrt{R_g} \qquad (1)$$

Example: Using $1^1/_2$ in. diameter cutter, determine the number of divisions to be used for a cam of $1^1/_2$ in. rise in 90°. The base radius is 2 in.

The average profile radius is $2 + 0.75 = 2.75$ in. Therefore

$$\Delta\theta = \frac{9}{2.75}\sqrt{0.75} = 2.82°$$

$$\frac{90}{2.82} = 30.2$$

The nearest integral factor of 120 is 30. Therefore every fourth point of the displacement factors would be used.

Profile Determination. As has been stated point-to-point positions of the trace point are determined by polar coordinates. Calculation and layout procedures will be demonstrated for the common types of open or closed cams with translating or swinging arm followers.

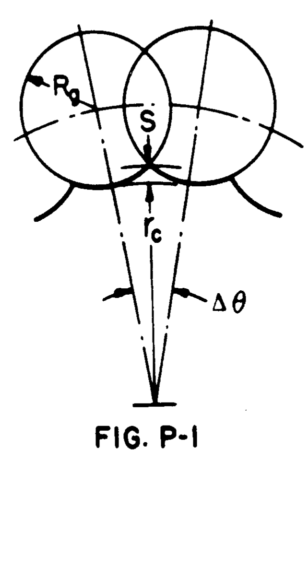


FIG. P-1

On-Center Translating Follower Nomenclature

h = total linear displacement of follower (in.)
y = linear displacement of follower at any point (in.)
β = total angular displacement of cam (deg.)
θ_n = angular displacement of cam at any point (deg.)
R_n = radius of any point on pitch curve from cam axis (in.)
R_o = minor radius (in.)
K = displacement factor
N = total number of divisions
n = point position

Calculation Procedure. See Fig. P-2. Point-to-point positions of the trace point are calculated from the following equations.

$$y = Kh \tag{2}$$

$$R_n = R_o + y \tag{3}$$

$$\theta_n = n\frac{\beta}{N} \tag{4}$$

EXAMPLE. Determine the point-to-point positions of the trace point of a cycloidal rise of 2 in. in 90°, with a minor radius of 3.5 in.

The results of applying Equations 2 and 3, using Table E-4 to evaluate the displacement factors, appear in Table P-1. Only six points are shown, which, of course, would not be sufficient in practice. It is recommended that calculations be presented in tabular form as shown.

Point	θ_n	K	y	R_n
0	0	0	0	3.5000
20	15	0.02844	0.0569	3.5569
40	30	0.19550	0.3910	3.8910
60	45	0.50000	1.0000	4.5000
80	60	0.80450	1.6090	5.1090
100	75	0.97166	1.9433	5.4433
120	90	1.00000	2.0000	5.5000

TABLE P-1

Layout Procedure. The tabulated data plus the follower diameter completely describe the cam contour. However, it is desirable to have the detail drawing show an accurate picture of the profile as depicted in Fig. P-3. The layout procedure for the detail is as follows.

1. With cam axis as center inscribe a circle with radius R_o.
2. Divide this circle into the required angular displacements (β) of rise and return and the dwells; the order to be opposite to that of cam rotation.
3. Divide the angular displacements (β) into N equal sections by radial lines.
4. With cam axis as center, inscribe radii R_n, intersecting the radial lines at corresponding angular d'splacements θ_n.
5. With each point, thus determined, as a center, inscribe a circle with radius of the roller follower.
6. Draw a smooth curve tangent to these circles to outline the cam profile.

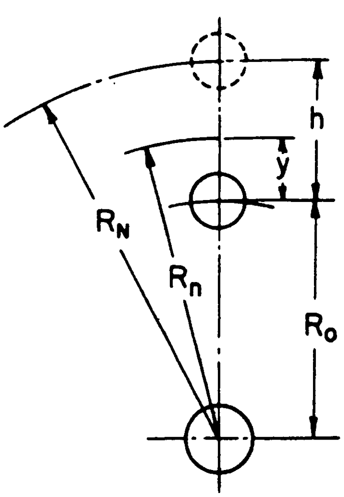


FIG. P-2

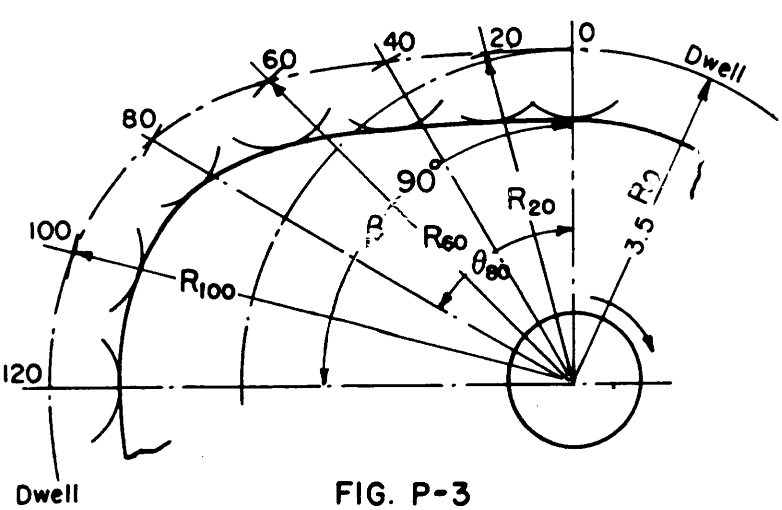


FIG. P-3

Non-Radial Followers. Calculations involving offset and swinging arm followers are more involved than those with an on-center follower. In Fig. P-2 it is seen that all points of the linear displacement lie on the same radial line. With the offset condition appearing in Fig. P-4, it may be seen that every point of the linear displacement lies on a different radial line. This condition also exists with swinging arm followers. To meet these conditions compensating calculations must be made.

Off-Set Translating Follower

Nomenclature

a, b = rectangular offsets of follower at minor radius position from cam axis (in.)
h = total linear displacement of follower (in.)
y = linear displacement of follower at any point (in.)
R_n = radius at any point on pitch curve (in.)
R_o = minor radius (in.)
K = displacement factor
N = total number of divisions
n = point position
β = total angular displacement of cam (deg.)
θ_n = angular displacement of cam at any point (deg.)
δ = angular offset of follower (deg.)
σ = adjustment angle (deg.)
ψ = cam angle (deg.)

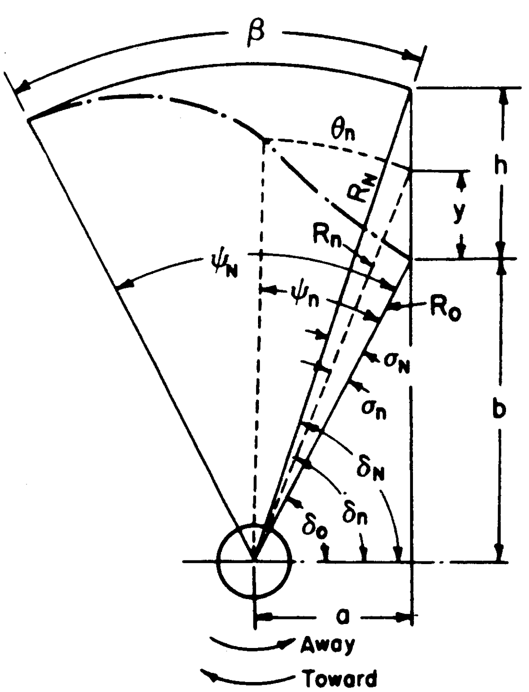


FIG. P-4
RISE - Rotating toward offset
RETURN - Rotating away from offset

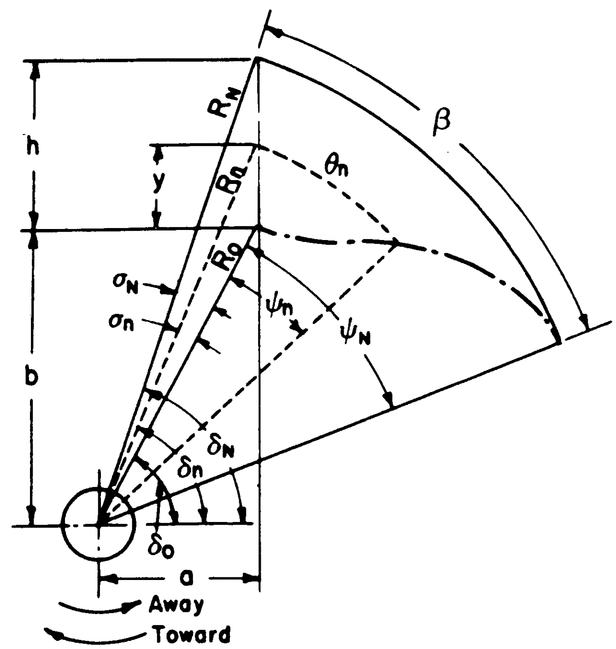


FIG. P-5
RISE - Rotating away from offset
RETURN - Rotating toward offset

Calculation Procedure. As indicated in Figs. P-4 and P-5, procedures differ slightly according to rise, return, and direction of rotation. The cam radii and angles are established by the following.

Determine a, b, h, β, and N from the required operation conditions and the geometry of the system.

$$y = Kh \tag{5}$$

$$\delta_n = \tan^{-1}\frac{b+y}{a} \tag{6}$$

$$R_n = \frac{b+y}{\sin\delta_n} \tag{7}$$

$$\sigma_n = \delta_n - \delta_o \tag{8}$$

$$\theta_n = \frac{n}{N}\beta \tag{9}$$

$$\psi_n = \theta_n + \sigma_n, \quad \text{Fig. P-4} \tag{10}$$

$$\psi_n = \theta_n - \sigma_n, \quad \text{Fig. P-5} \tag{11}$$

In Fig. P-6 is shown a convenient form for sequence of computation and record of calculated data.

Machine ______	Cam ______	Type Curve ______
$a=$ ______	$h=$ ______	$N=$ ______
$b=$ ______	$\beta=$ ______	$\beta/N=$ ______

Point		Kh	$b+y$	$\frac{b+y}{a}$			$\frac{b+y}{\sin\delta_n}$	$\delta_n-\delta_o$	$n\frac{\beta}{N}$	$\theta_n \pm \sigma_n$
n	K	y		$\tan\delta_n$	δ_n	$\sin\delta_n$	R_n	σ_n	θ_n	ψ_n
0										
1										
2										
N										

FIG. P-6

Layout Procedure. Points on the pitch curve are established by radii R_n and cam angles ψ_n as shown in Fig. P-7.

1. With cam axis as center inscribe a circle with radius R_o.
2. Divide this circle into the calculated cam angles ψ_N of rise and return and the dwells, the order to be opposite to that of cam rotation.
3. Divide cam angles ψ_N into N sections by radial lines at angles ψ_n, measured from radial line R_o.
4. With cam axis as center, inscribe radii R_n, intersecting corresponding radial lines at angles ψ_n.
5. With each point, thus determined, as center inscribe a circle with radius of the roller follower.
6. Draw a smooth curve tangent to these circles to outline the cam profile.

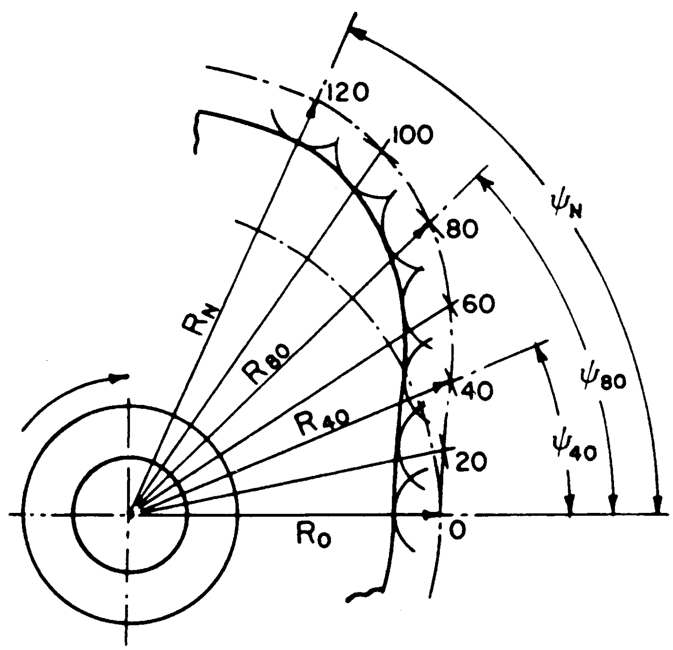


FIG. P-7

References

1. Which Way to Make a Cam. H. A. Rothbart. Prod. Eng. Mar. 3, 1958.
2. Cams—Design, Dynamics and Accuracy. H. A. Rothbart. John Wiley & Sons, Inc. 1956.
3. Cam Design. W. D. Cram. Machine Design. Nov. 1, 1956.

Swinging Arm Follower

Nomenclature

a = distance, cam axis to pivot (in.)
b = length of swinging arm (in.)
e, f = offsets of pivot from cam axis (in.)
R_n = radius at any point on pitch curve (in.)
R_o = minor radius (in.)
K = displacement factor
N = total number of divisions
n = point position
β = total angular displacement of cam (deg.)
θ_n = angular displacement of cam at any point (deg.)
ϕ = angular displacement of swinging arm (deg.)
ρ = angular offset of b from a (deg.)
δ = angular offset of follower (deg.)
σ = adjustment angle (deg.)
ψ = cam angle (deg.)

Calculation Procedure. As shown in Figs. P-8 through P-11 various conditions are possible, depending on relative position of pivot to cam axis, rise or return and cam rotations. Cam angles and radii are established by the following procedure.

Determine R_o, b, e, f, N, ϕ_N and β from the required operation conditions and the geometry of the system.

$$a = \sqrt{e^2 + f^2} \tag{12}$$

$$\rho_o = \cos^{-1} \frac{a^2 + b^2 - R_o^2}{2ab} \tag{13}$$

$$\phi_n = K\phi_N \tag{14}$$

$$\rho_n = \rho_o + \phi_n \tag{15}$$

$$R_n = \sqrt{a^2 + b^2 - 2ab \cos \rho_n} \tag{16}$$

$$\delta_n = \sin^{-1} \frac{b \sin \rho_n}{R_n} \tag{17}$$

$$\theta_n = \frac{n}{N}\beta \tag{18}$$

$$\sigma_n = \delta_0 - \delta_n \tag{19}$$

$$\psi_n = \theta_n + \sigma_n, \quad \text{Fig. P-8 and P-11} \tag{20}$$

$$\psi_n = \theta_n - \sigma_n, \quad \text{Fig. P-9 and P-10} \tag{21}$$

In Fig. P-12 is shown a convenient form for sequence of computation and record of calculated data.

Layout Procedure. The layout procedure is identical to the offset-translating follower.

Machine ______ Cam ______ Type Curve ______

a= ______ a^2+b^2= ______ β= ______
b= ______ ϕ_N= ______ N= ______
$2ab$= ______ ρ_o= ______ β/N= ______

Point		$K\phi_N$	$\rho_o + \phi_n$		$2ab \cos \rho_n$	$a^2+b^2-2ab \cos \rho_n$	$\sqrt{a^2+b^2-2ab \cos \rho_n}$		$b \sin \rho_n$	$\frac{b \sin \rho_n}{R_n}$		$\delta_o - \delta_n$	$\delta_n - \delta_o$	$n\frac{\beta}{N}$	$\theta_n \pm \sigma_n$
n	K	ϕ_n	ρ_n	$\cos \rho_n$			R_n	$\sin \rho_n$		$\sin \delta_n$	δ_n	σ_n		θ_n	ψ_n
0															
1															
2															
N															

FIG. P-12

When δ_o is greater than or equal to δ_N

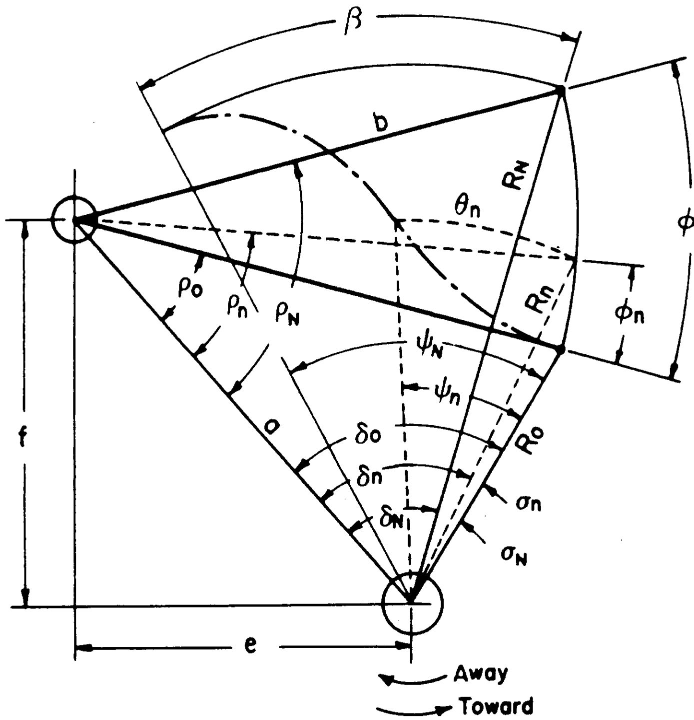


FIG. P-8
RISE - Rotating away from pivot
RETURN - Rotating toward pivot

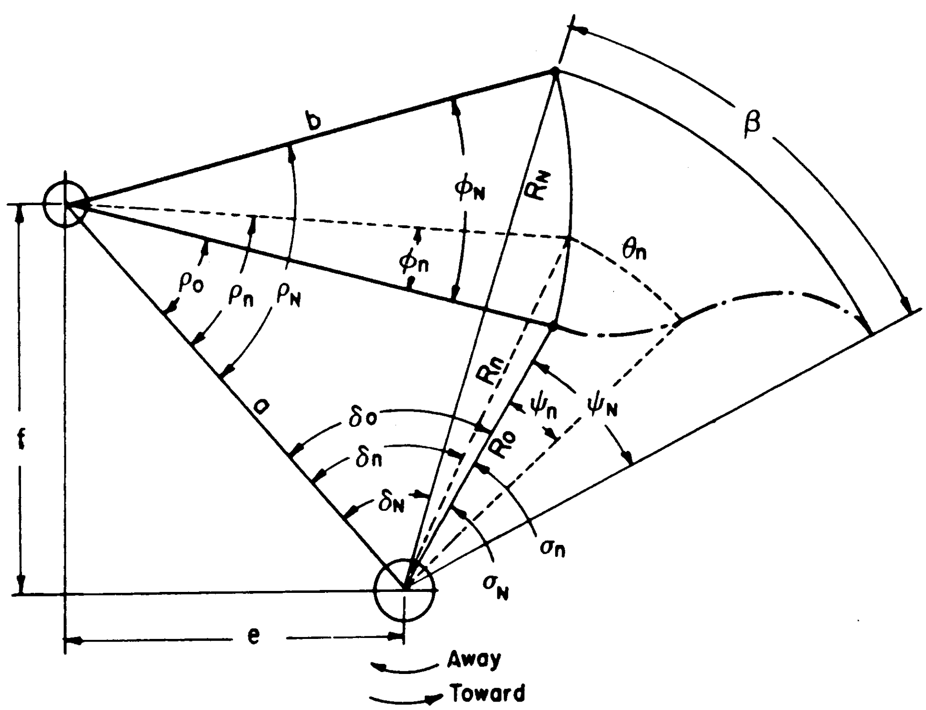


FIG. P-9
RISE - Rotating toward pivot
RETURN - Rotating away from pivot

When δ_0 is less than δ_N

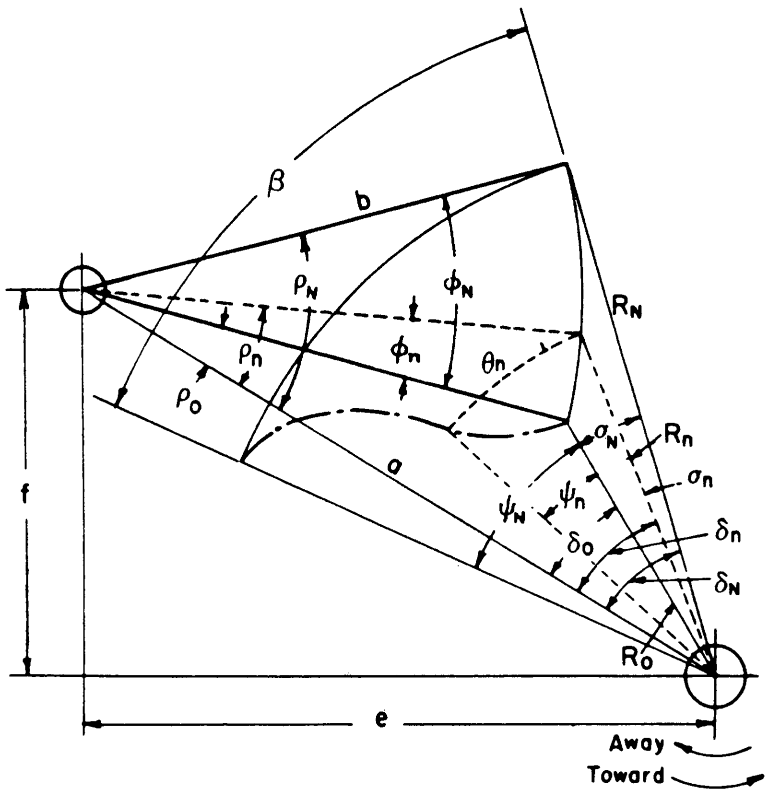


FIG. P-10
RISE - Rotating away from pivot
RETURN - Rotating toward pivot

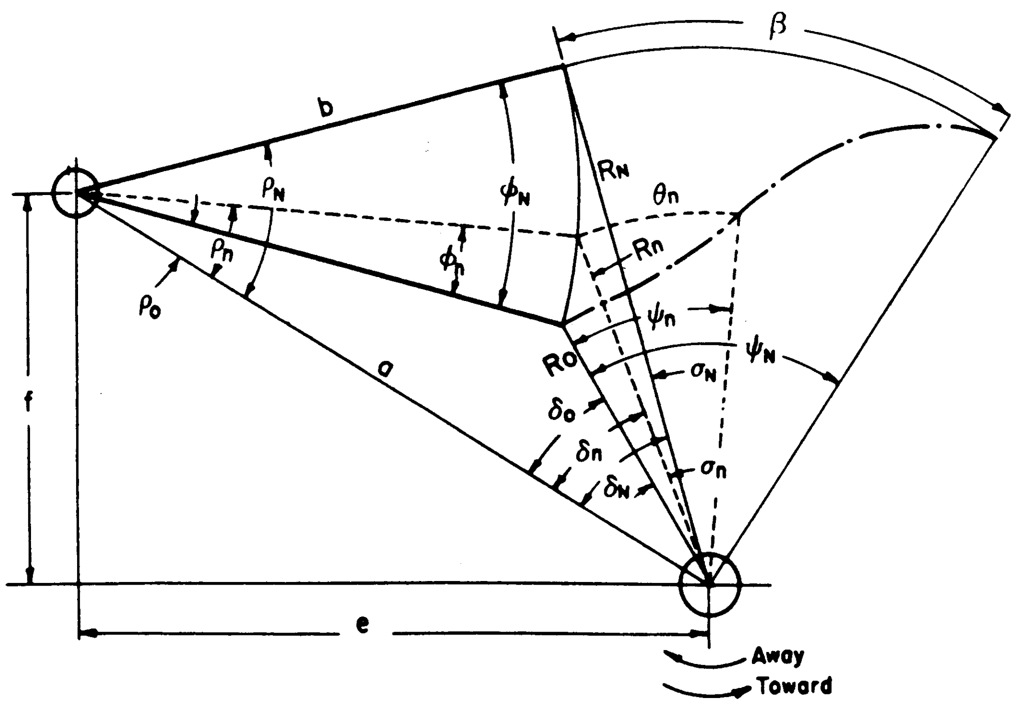


FIG. P-11
RISE - Rotating toward pivot
RETURN - Rotating away from pivot

Cylindrical Cams

Introduction. In Section B, Fig. 11-B was shown a cylindrical cam with the roller follower operating in a groove cut on the periphery of a cylinder. This is the most common type. Usually a cylindrical follower is used, but as the outer edge of the follower travels faster than the inner, some skidding is present. To decrease the velocity difference, it is advisable to keep the width of the roller as narrow as possible, consistent with strength considerations. Sometimes a tapered roller is utilized to compensate for the difference. In this case the vertex of the roller should coincide with the center of the cam shaft. The follower may be of the translating or swinging arm type. Generally calculation and layout procedures are simpler than those for the radial disk cam.

Cylindrical Cam—Translating Follower. See Fig. R-1. The first step is determination of the cam diameter. As in the disk cam, the significant parameters are maximum pressure angle and the least radius of curvature.

Pressure Angle. Pressure angles may be calculated from the following equation:

$$\tan \gamma = \frac{57.3 C_v h}{R\beta} \qquad (1)$$

in which R is the outside diameter of the cam, the other symbols as in previous nomenclature. The maximum pressure angle should be approximately 30°. It occurs at the transition point. Note that Equations 3, 4, and 5 from Section F, and Equations 16, 17, and 18 from Section K are applicable.

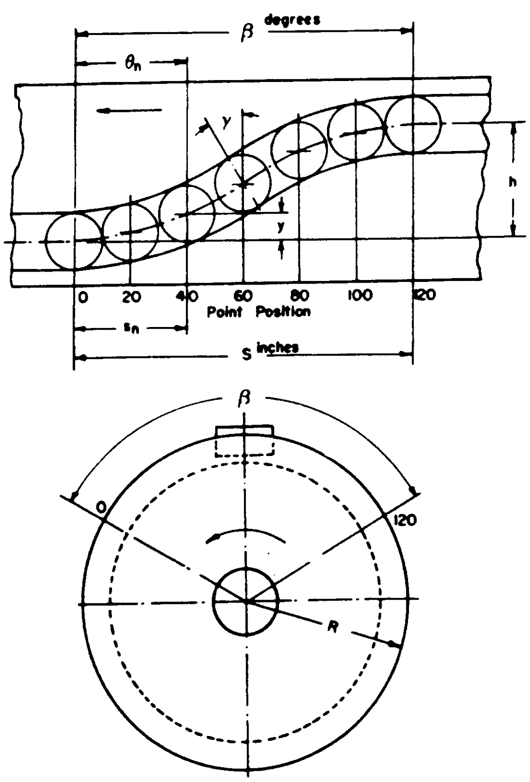


FIG. R-1

Radius of Curvature. The radius of curvature can be approximated in a manner similar to that used for a disk cam.

In Fig. R-2, let A and B be two adjacent points of a very small section of the cam pitch curve. Applying the following equations will give a close approximation of the radius of curvature.

$$\Delta s = \frac{\pi R \beta}{180 N} \qquad (2)$$

$$\Delta y = y_B - y_A \qquad (3)$$

$$R_c = \frac{\sqrt{\Delta s^2 + \Delta y^2}}{\sin(\gamma_B - \gamma_A)} \qquad (4)$$

where β = total angular displacement of cam, N = number of divisions, and the other symbols as indicated in Fig. R-2.

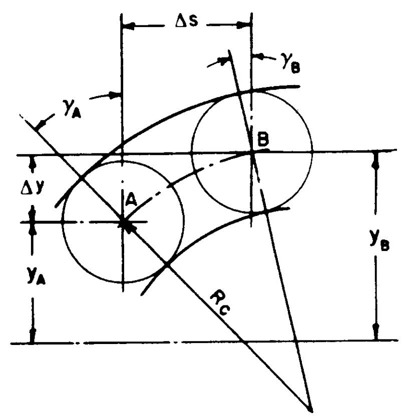


FIG. R-2

Calculation Procedure. The cam contour may be constructed on a sheet metal or paper template and transferred directly by wrapping around the cam blank, and machining to a scribed line. This, of course, is a very inaccurate method. The necessary dimensions for this method are the intermediate displacements y and the length of the arcs s_n, which are indicated in Fig. R-1.

The more accurate method consists of constructing a flat, radial master cam which controls the cutting of the cam contour on a commercial cam miller. The following equations are applicable to the profile determination; in which n = point position, N = number of divisions; the other symbols as indicated in Fig. R-1.

$$y = Kh \qquad (5)$$

$$\theta_n = \frac{n}{N}\beta \qquad (6)$$

$$S = \frac{2\pi R\beta}{360} \qquad (7)$$

$$s_n = \frac{S}{N} \qquad (8)$$

Layout Procedure. Points on the pitch curve are established by horizontal dimension s_n and vertical dimensions y.

1. Draw a straight line equal to circumference of cam.
2. Divide this line into sections S corresponding to the angular displacement and the dwells; the order to be opposite that of cam rotation.
3. Divide each section S into N equal parts by vertical lines.
4. On each line establish corresponding displacement y as calculated from Equation 5.
5. With each point, thus determined, inscribe a circle with radius of the follower.
6. Draw smooth curves tangent to these circles to outline the cam profile.

Layout Procedure.

1. Draw a straight line equal to circumference of cam.
2. Divide this line into sections S, corresponding to the angular displacement β and the dwells, the order to be opposite that of cam rotation.
3. Divide each section S into N equal parts.
4. With each point as center inscribe a radius b.
5. From a line through lowest point, establish corresponding displacements y, on each arc.
6. With each point, thus determined, as center inscribe a circle with radius of follower.
7. Draw smooth curves tangent to these circles to outline the cam profile.

Swinging Arm Follower. For commercial cam millers the master cam is a flat disk. A swinging arm attachment compensates for the offset of the follower. The profile is established from the calculated intermediate displacements y and the length of the swinging arm b.

Calculation Procedure. The following equations, applied to Fig. R-3, establish the profile

$$\phi_n = K\phi_N \qquad (9)$$

$$y = b[\sin(0.5\phi_N) - \sin(0.5\phi_N - \phi_n)] \qquad (10)$$

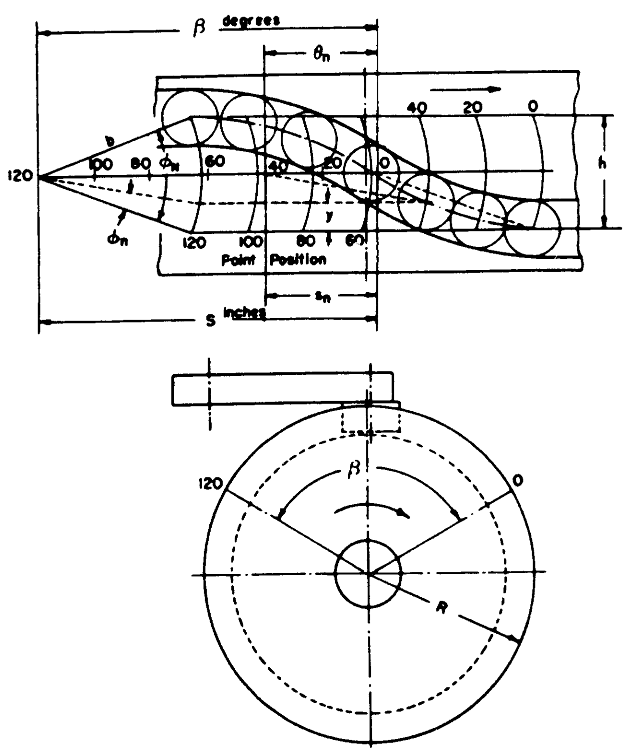


FIG. R-3

Indexing Cams. Cams for intermittent motions, such as indexing a dial or a turret, are a special form of cylindrical cam. They are available commercially for various ranges of indexing stations and proportions of motion and dwell. They have an advantage over the well-known Geneva motion in that controlled acceleration can be obtained by using the basic curves.

In Fig. R-4 is shown a development of an indexing cam of a type manufactured by Standard Tool and Manufacturing Co., Arlington, N. J. It uses tapered roller followers, which are locked during the dwell by the tapered sides of a raised wedge on the periphery of the cam.

Adjustment for minimum backlash and wear is accomplished by shimming the dial. This type can be made with cylindrical followers but then requires a locking-pin for accurate indexing.

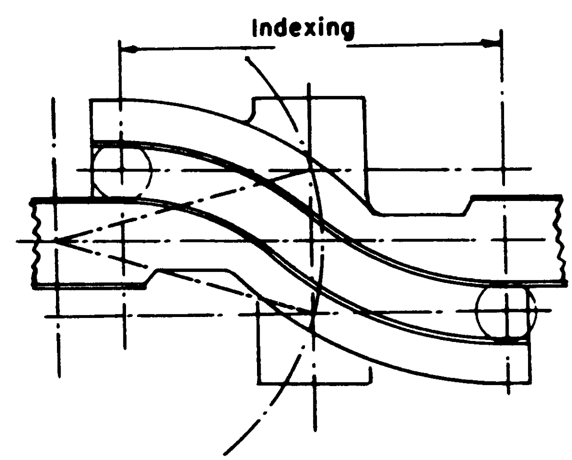


FIG. R-4

Figure R-5 illustrates a type made by Commercial Cam Division, Wheeling, Ill. During the dwell period, the cylindrical rollers are locked between two hardened and ground steel plates mounted on the ends of the cam.

The type shown in Fig. R-6 has the trade name of "Roller gear." The followers are anti-friction ball bearings, and are locked by the raised wedge. Provision for adjusting the distance between the centers of cam and dial must be made to minimize backlash and compensate for wear.

Further information may be obtained from the catalogs of the respective companies.

Multi-groove cams are sometimes used to reduce the width of the cam. The dial is equipped with twice the number of followers as there are indexing stations. For information on this subject, see Reference 4 at end of the section.

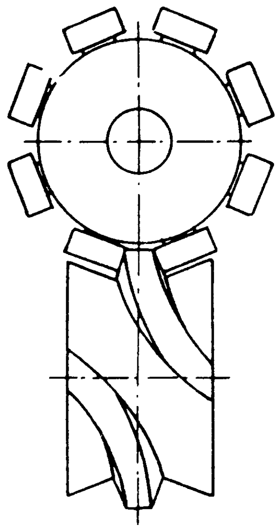


FIG. R-6

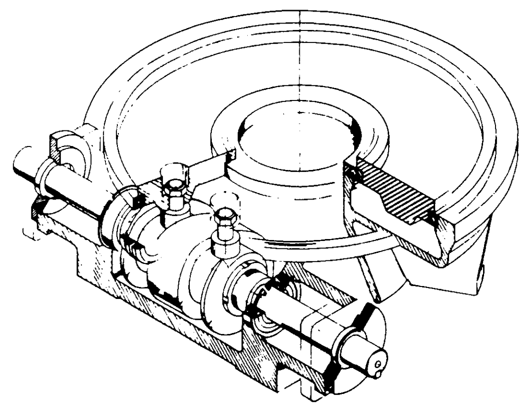
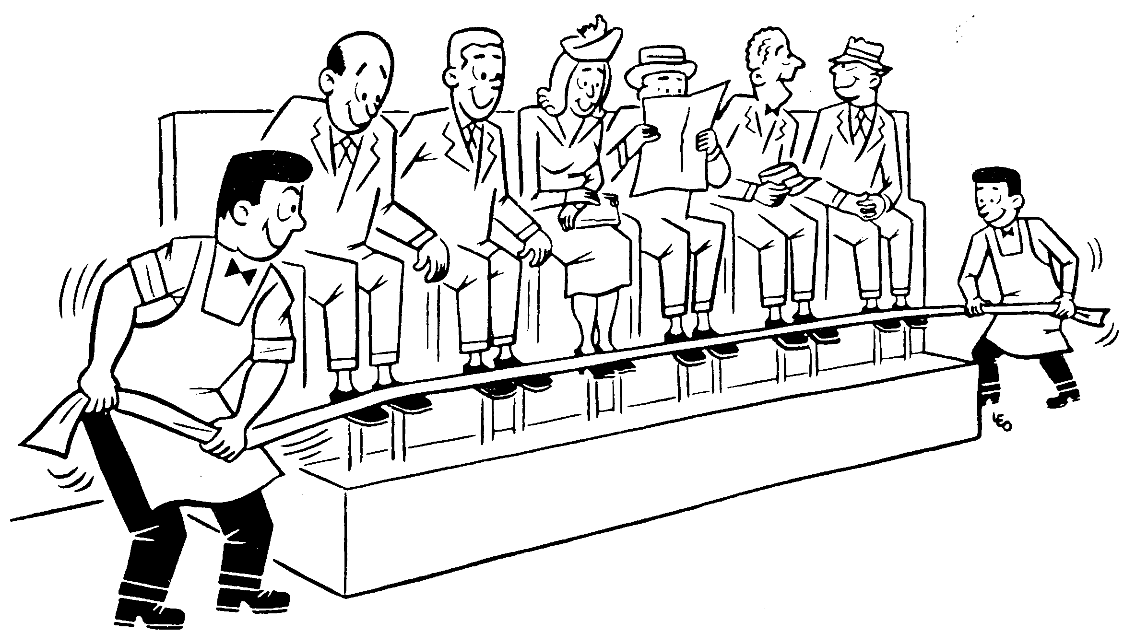


FIG. R-5



INTERMITTENT MOTION

Screw-Type Indexing Cam. This cam combines some of the features of the mechanisms shown in Figs. 4 and 6. The motion is imparted to two tapered followers by a wedge shaped rib. The continuation of this rib locks the dial during the dwell period.

In Fig. R-7 is shown the single roller type; in Fig. R-8 the dual roller type. Detailed calculation procedures are presented for each.

Nomenclature

c = center distance—dial to cam shaft (in.)
d = distance—primary curve origin to secondary curve origin (in.)
g = offset of follower from dial center at origin of secondary curve (in.)
h = offset of follower from dial center at origin of primary curve (in.)
r = radius—dial center to follower center (in.)
y = linear displacement of follower from origin at any point of primary curve (in.)
y' = linear displacement of follower from origin at any point of secondary curve (in.)
n = point position
N = number of division points
K = displacement factor
F = number of followers
X = number of indexing stations
β = total angular displacement of cam (deg.)
ϕ = angular displacement of follower (deg.)
α = angular offset of follower from dial center (deg.)

Calculation Procedure—Single Roller. See Fig. R-7. Determine c, X, and β from required conditions and geometry of the system. Apply the following equations to determine the profile.

$$F = X \tag{11}$$

$$\phi_N = \frac{360}{X} \tag{12}$$

$$r = \frac{c}{\cos 0.5\phi_N} \tag{13}$$

$$h = r \sin 0.5\phi_N \tag{14}$$

$$g = r \sin 1.5\phi_N \tag{15}$$

$$d = c - r \cos 1.5\phi_N \tag{16}$$

$$\phi_n = K\phi_N \tag{17}$$

For Curve A-B ($\phi_n < 0.5\phi_N$)

$$\alpha_n = 0.5\phi_N - \phi_n \tag{18}$$

$$y = h - r \sin \alpha_n \tag{19}$$

For Curve B-C ($\phi_n > 0.5\phi_N$)

$$\alpha_n = \phi_n - 0.5\phi_N \tag{20}$$

$$y = h + r \sin \alpha_n \tag{21}$$

For Curve A′-B′

$$\alpha_n = 1.5\phi_N - \phi_n \tag{22}$$

$$y' = g - r \sin \alpha_n \tag{23}$$

Calculation Procedure—Dual Roller. See Fig. R-8. Determine c, X, and β from required conditions and geometry of the system. Apply the following equations to determine the profile.

$$F = 2X \tag{24}$$

$$\phi_N = \frac{360}{X} \tag{25}$$

$$r = \frac{c}{\cos 0.25\phi_N} \tag{26}$$

$$h = r \sin 0.25\phi_N \tag{27}$$

$$g = r \sin 0.75\phi_N \tag{28}$$

$$d = c - r \cos 0.75\phi_N \tag{29}$$

$$\phi_n = K\phi_N \tag{30}$$

For Curve A-B ($\phi_n < 0.25\phi_N$)

$$\alpha_n = 0.25\phi_N - \phi_n \tag{31}$$

$$y = h - r \sin \alpha_n \tag{32}$$

For Curve B-C ($\phi_n > 0.25\phi_N$)

$$\alpha_n = \phi_n - 0.25\phi_N \tag{33}$$

$$y = h + r \sin \alpha_n \tag{34}$$

For Curve A′-B′ ($\phi_n < 0.75\phi_N$)

$$\alpha_n = 0.75\phi_N - \phi_n \tag{35}$$

$$y' = g - r \sin \alpha_n \tag{36}$$

For Curve B′-C′ ($\phi_n > 0.75\phi_N$)

$$\alpha_n = \phi_n - 0.75\phi_N \tag{37}$$

$$y' = g + r \sin \alpha_n \tag{38}$$

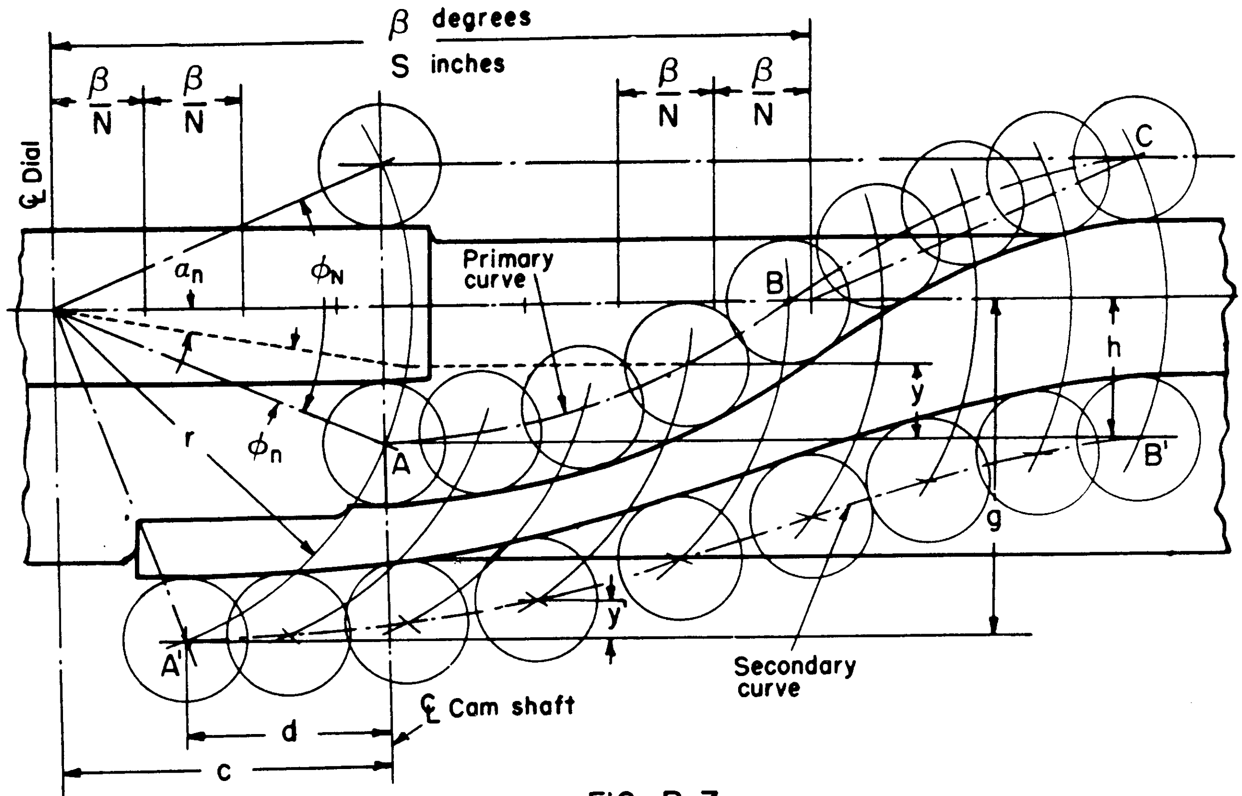


FIG. R-7

Single Roller

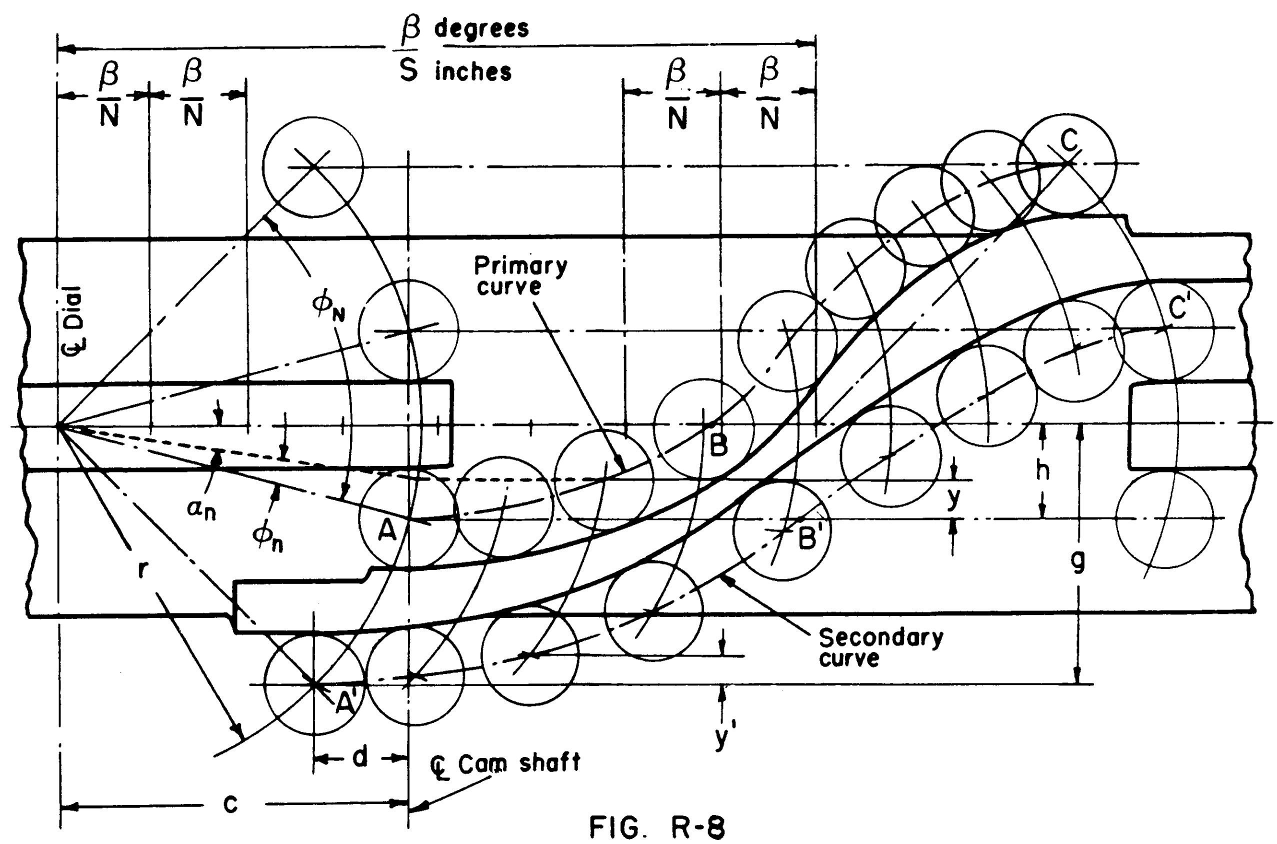


FIG. R-8

Dual Roller

Size Determination. The main factors determining the cam diameter are the offset d and the minimum thickness of the ridge.

As illustrated in Fig. R-9, the smaller the radius of the cam, the less contact there will be between the cam profile and follower at the origin A′ of the secondary curve. To provide point contact only, the cam radius must equal P as shown by the dot and dash line. Therefore, to provide line contact

$$R > \frac{d^2 + f^2}{2f} \tag{39}$$

in which d = origin of primary to origin of secondary curve (in.), and f = effective length of follower (in.).

To contact a definite proportion of the effective length of the follower

$$R = \frac{d^2 + f^2(1 - z)^2}{2f(1 - z)} \tag{40}$$

where z = proportional contact.

For a given radius, the proportional contact is

$$z = 1 - \frac{R - \sqrt{R^2 - d^2}}{f} \tag{41}$$

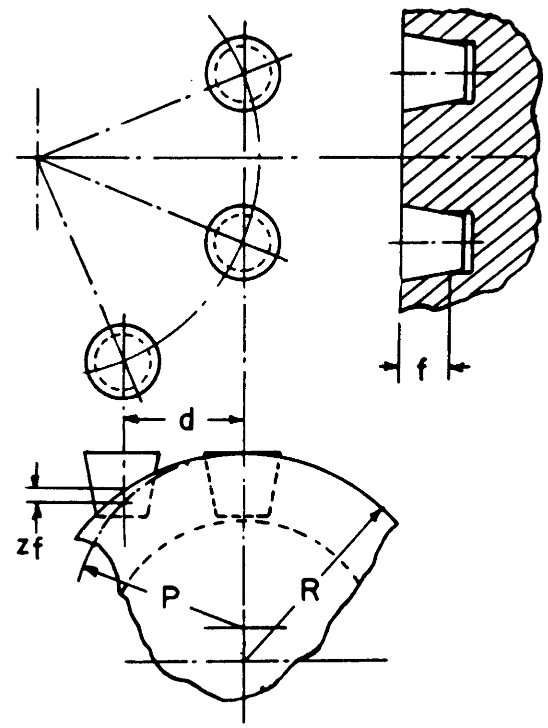
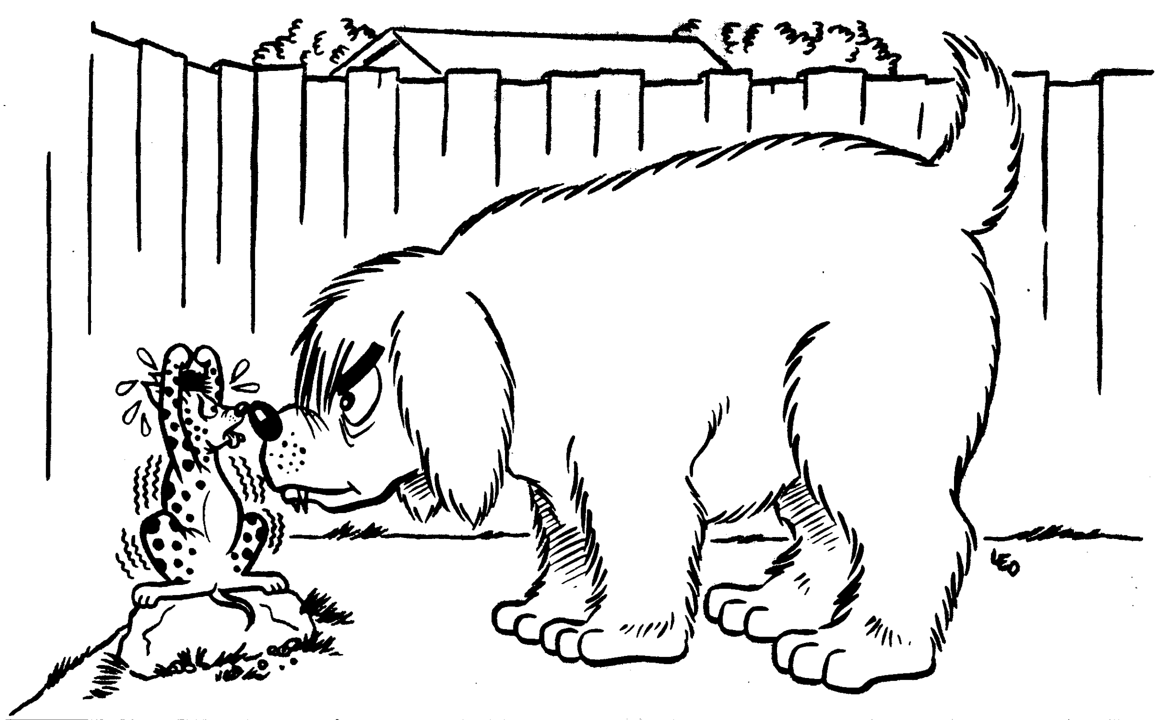


FIG. R-9



SIZE DETERMINATION

Ridge Thickness. Determining the minimum ridge thickness would involve much calculation. However, the following layout method will give the approximate thickness and may save the time and labor of back tracking if a specified radius and indexing period do not provide a minimum thickness of sufficient strength. See Fig. R-10.

1. Draw line 0-1 equal to $\pi R\beta/180$.
2. With point 1 as center, inscribe r, locating point 2.
3. Erect perpendicular at mid-point of line 2-3, intersecting extension of line 3-4 at point 5.
4. With point 5 as center, inscribe radius equal to line 5-3.
5. With point 2 as center, inscribe radius equal to line 3-6, locating point 7.
6. Erect perpendicular at mid-point of line 7-6, intersecting at point 8 a line through point 6 parallel to line 3-5.
7. With point 8 as center inscribe radius equal to line 8-6.
8. Scale M (which is the approximate distance between follower centers) on extension of line through points 8 and 5.

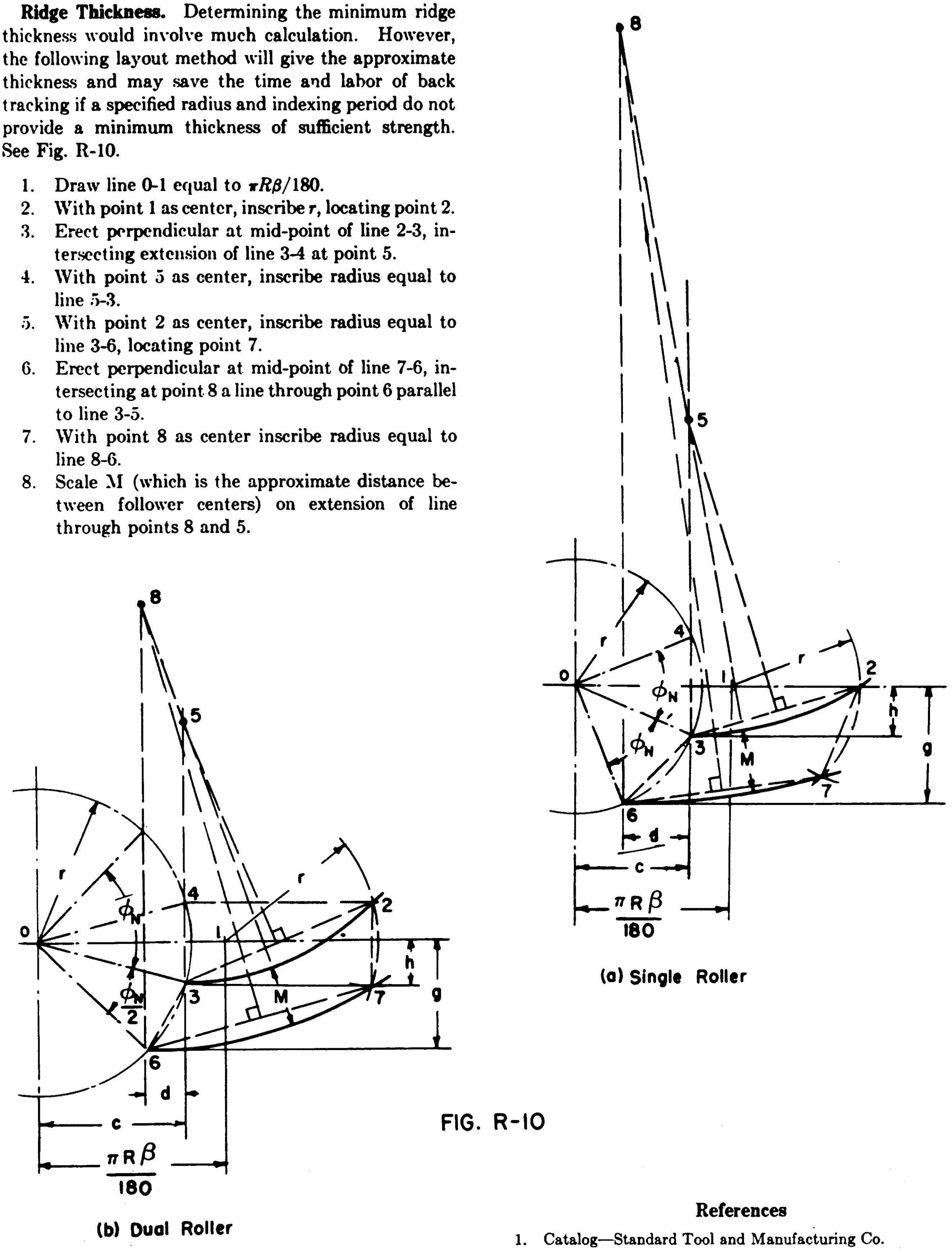


FIG. R-10

References

1. Catalog—Standard Tool and Manufacturing Co.
2. Catalog—Commercial Cam and Machine Co.
3. Catalog—Ferguson Machine Co.
4. Mechanisms for Intermittent Motion. Otto Lichtwitz. Machine Design. March, 1952.

Effective Weight of Cam Follower Systems

Typical Systems. In many cam systems, there is some sort of linkage between the cam and the end element which is being moved. In determining the forces on the cam the effect of the linkage should be considered.

In Figs. S-1 through S-3 are shown simple, although typical, cam systems, consisting of the cam, push rod, lever, and the end element. The total effective weight of the follower system is the effective weight of the rod, lever, and end element. Not all of these is the actual weight of the parts.

The effective weight of the push rod involves its kinetic energy, which is

$$E_K = \frac{1}{2}\frac{W_P}{g}v_p^2 \qquad (1)$$

where W_P = weight of rod; v_p = velocity of rod, and g = gravity constant (386 in./sec.²). The corresponding energy on the end element side must be the same, so

$$E_K = \frac{1}{2}\frac{W_{EP}}{g}v_M^2 \qquad (2)$$

where W_{EP} = effective weight of the push rod; v_M = velocity of the end element.

Equating these two expressions and solving,

$$W_{EP} = W_P\left(\frac{v_p}{v_M}\right)^2 \qquad (3)$$

Let R_v be the velocity ratio of the arms, then

$$R_v = \frac{v_M}{v_p} = \frac{a}{b} \qquad (4)$$

and

$$W_{EP} = \frac{W_P}{R_v^2} \qquad (5)$$

If the rod is long and slender, flexibility will prevent its entire mass being accelerated at the same rate. It is usual practice to assume that only one third of the weight is affected by the acceleration. Therefore for long slender rods,

$$W_{EP} = \frac{W_P}{3R_v^2} \qquad (6)$$

The effective weight of the lever is derived from the moment of inertia. The moment of inertia about the center of oscillation is

$$I_o = I_g + mc^2 \qquad (7)$$

where I_g = moment of inertia about the center of gravity; m = mass of the lever; and c = distance between centers of oscillation and gravity. But I_o may also be written as if the effective weight (W_{EL}) were concentrated at a distance "a" from the center of oscillation.

$$I_o = \frac{W_{EL}a^2}{g} \qquad (8)$$

and

$$W_{EL} = \frac{I_o a^2}{g} \qquad (9)$$

An approximation of the effective weight of the lever (assuming a fairly uniform section) may be determined from the following equations:

For Fig. S-1

$$W_{EL} = \frac{2W_L}{5}\left(\frac{1}{R_v^2} - \frac{1}{R_v} + 1\right) \qquad (10)$$

For Fig. S-2

$$W_{EL} = \frac{2W_L}{5} \qquad (11)$$

For Fig. S-3

$$W_{EL} = \frac{2W_L}{5R_v^2} \qquad (12)$$

where W_L = actual weight of the lever; and R_v = a/b = velocity ratio.

The effective weight (W_M) of the end element is its actual weight.

The effective weight of the entire follower system referred to the end element side is

$$W_E = W_{EP} + W_{EL} + W_M \qquad (13)$$

Thus, the system is equivalent to one moving a weight W_E by means of a weightless linkage.

The forces on the cam (weight and inertial) are obtained by multiplying the corresponding end element forces by the velocity ratio (R_v).

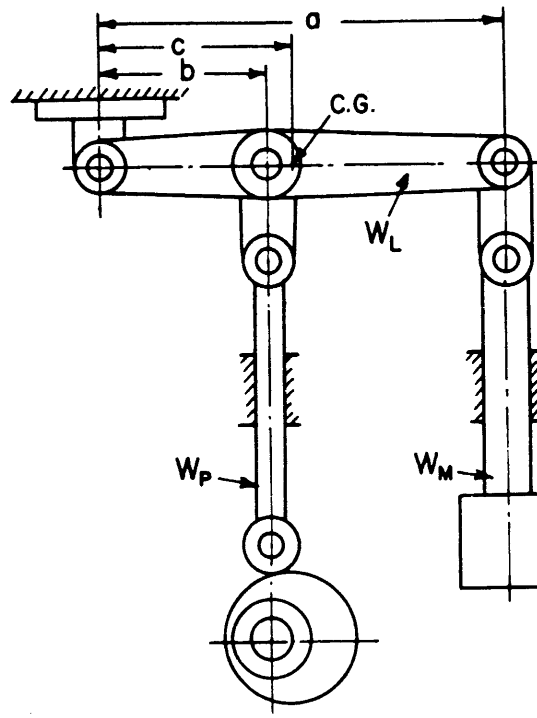


FIG. S-2

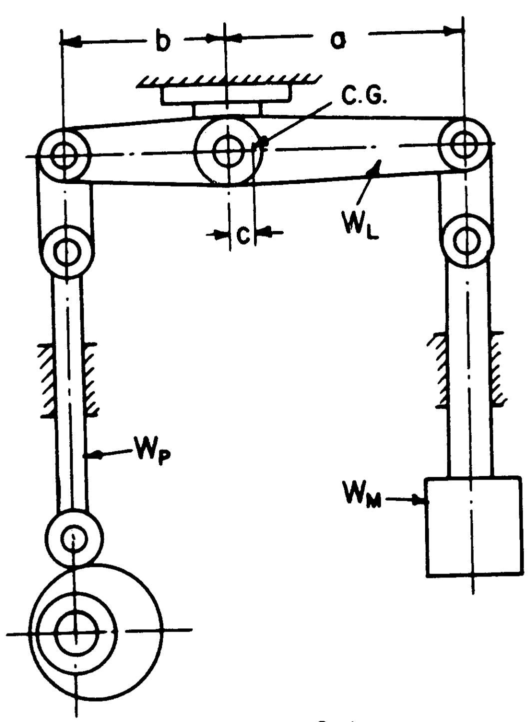


FIG. S-1

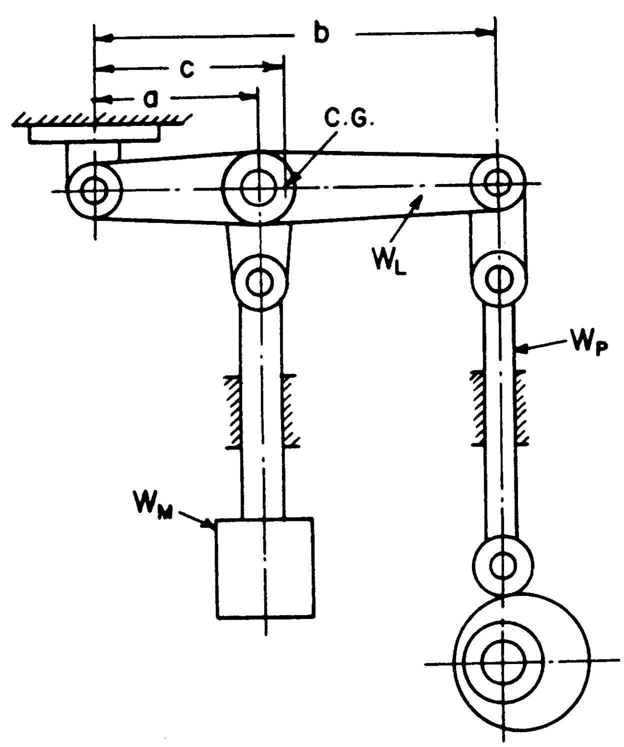


FIG. S-3

Polynomial Cam Curves

WHEN the function of a cam is simply to move a follower through a definite displacement in a prescribed time, design requirements usually can be satisfied by one of the basic curve forms: Cycloidal, harmonic, parabolic, etc. Final choice will depend only on the type of motion which is judged to be best for the particular event.

When stringent terminal or intermediate conditions are imposed, the basic curves require considerable manipulation to achieve the required objectives. Usually some compromise must be reached with resultant deterioration of desirable features.

Another possibility is to resort to a polynomial curve, tailored to satisfy the specific problem requirements. However, if more than six conditions are involved, a polynomial of high order will result. Without high-speed computing equipment, the mathematics becomes tedious and time-consuming.

This article presents a method of combining low-power polynomials to meet specified conditions. Mathematical operations are relatively simple, and can be easily and quickly handled with slide rule and desk calculator. The procedures outlined will produce continuous and finite velocity, acceleration, and pulse (jerk) curves.

Design Equations

Consider any cam-displacement curve where the event takes place in time t. Let original displacement $y_O = 0$ and final displacement $y_F = h$. Typical displacement diagrams appear in Fig. 1.

The general polynomial[1] to accommodate six conditions is:

$$y = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 \qquad (1)$$

Differentiating this equation successively with respect to time, evaluating constants, and collecting terms give the equations for displacement (y), velocity (dy/dt), acceleration (d^2y/dt^2), and pulse (d^3y/dt^3). These lengthy equations may be simplified to:

$$y = H_1h + H_2v_Ot + H_3v_Ft + H_4a_Ot^2 + H_5a_Ft^2 \qquad (2)$$

$$\frac{dy}{dt} = V_1\frac{h}{t} + V_2v_O + V_3v_F + V_4a_Ot + V_5a_Ft \qquad (3)$$

$$\frac{d^2y}{dt^2} = A_1\frac{h}{t^2} + A_2\frac{v_O}{t} + A_3\frac{v_F}{t} + A_4a_O + A_5a_F \qquad (4)$$

$$\frac{d^3y}{dt^3} = P_1\frac{h}{t^3} + P_2\frac{v_O}{t^2} + P_3\frac{v_F}{t^2} + P_4\frac{a_O}{t} + P_5\frac{a_F}{t} \qquad (5)$$

where v_O = original velocity, v_F = final velocity, a_O = original acceleration, a_F = final acceleration, and

$$H_1 = 10x^3 - 15x^4 + 6x^5$$

$$H_2 = x - 6x^3 + 8x^4 - 3x^5$$

$$H_3 = -4x^3 + 7x^4 - 3x^5$$

$$H_4 = \frac{x^2}{2} - \frac{3x^3}{2} + \frac{3x^4}{2} - \frac{x^5}{2}$$

$$H_5 = \frac{x^3}{2} - x^4 + \frac{x^5}{2}$$

Corresponding expressions for coefficients V, A, and P may be determined directly from these expressions for H by successive differentiation. For example, $V_1 = 30x^2 - 60x^3 + 30x^4$, $A_1 = 60x - 180x^2 + 120x^3$, and $P_1 = 60 - 360x + 360x^2$.

Values of coefficients H, V, A, and P are tabulated in Tables 1 through 4 for the range from $x = 0$ to $x = 1$. Table 1 (displacement) provides for 60 divisions, which will be the basis of reference for all intermediate points. For example, in a curve between terminal points A and B, the 24th point ($x = 0.4$) will be identified as point AB-24.

Equations 2 through 5 display some interesting characteristics:

1. If v_O, v_F, a_O and a_F are zero, the acceleration is a sine curve, similar to the cycloidal.
2. If $v_O = 0$, $v_F = 0$, $a_O = \pi^2h/2t^2$, and $a_F = -\pi^2h/2t^2$, the acceleration is a cosine curve, similar to the harmonic.
3. If $v_O = v_F = h/t$, and a_O and a_F are zero, the motion is constant velocity.
4. If $v_O = 0$, $v_F = 2h/t$, and $a_O = a_F = 2h/t^2$, result is a positive constant-acceleration curve.
5. If $v_O = 0$, $v_F = 2h/t$, and $a_O = a_F = -2h/t^2$, result is a negative constant-acceleration curve.

Thus, the characteristics of all of the common basic curves are embodied within one set of equations. Application of the tables and equations to the development of suitable cam curves for certain fixed conditions will be shown by specific examples.

[1]References are tabulated at end of article.

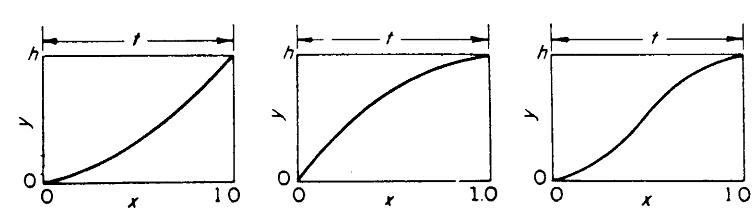


Fig. 1—Typical cam-displacement diagrams where event takes place in time t.

Table 1—Displacement Factors

Pt.	x	H_1	H_2	H_3	H_4	H_5
0	0	0	0	0	0	0
1		0.000045	0.016639	−0.000018	0.000132	0.000002
2		0.000352	0.033121	−0.000140	0.000502	0.000017
3	0.05	0.001158	0.049299	−0.000457	0.001072	0.000056
4		0.002675	0.065043	−0.001051	0.001807	0.000129
5		0.005088	0.080235	−0.001989	0.002674	0.000243
6	0.10	0.008560	0.094770	−0.003330	0.003645	0.000405
7		0.013230	0.108556	−0.005120	0.004691	0.000620
8		0.019216	0.121513	−0.007396	0.005786	0.000890
9	0.15	0.026612	0.133572	−0.010184	0.006909	0.001219
10		0.035494	0.144676	−0.013503	0.008038	0.001607
11		0.045917	0.154777	−0.017362	0.009153	0.002055
12	0.20	0.057920	0.163840	−0.021760	0.010240	0.002560
13		0.071521	0.171837	−0.026691	0.011282	0.003121
14		0.086724	0.178750	−0.032140	0.012267	0.003733
15	0.25	0.103516	0.184570	−0.038086	0.013184	0.004395
16		0.121869	0.189298	−0.044500	0.014022	0.005099
17		0.141742	0.192939	−0.051348	0.014775	0.005841
18	0.30	0.163080	0.195510	−0.058590	0.015435	0.006615
19		0.185817	0.197031	−0.066182	0.015998	0.007414
20		0.209876	0.197531	−0.074074	0.016461	0.008230
21	0.35	0.235169	0.197043	−0.082213	0.016821	0.009057
22		0.261599	0.195609	−0.090541	0.017077	0.009887
23		0.289060	0.193271	−0.098997	0.017230	0.010710
24	0.40	0.317440	0.190080	−0.107520	0.017280	0.011520
25		0.346619	0.186089	−0.116042	0.017230	0.012307
26		0.376474	0.181356	−0.124497	0.017084	0.013064
27	0.45	0.406873	0.175942	−0.132815	0.016845	0.013783
28		0.437685	0.169908	−0.140926	0.016519	0.014454
29		0.468773	0.163322	−0.148762	0.016110	0.015071
30	0.50	0.500000	0.156250	−0.156250	0.015625	0.015625
31		0.531227	0.148762	−0.163322	0.015071	0.016110
32		0.562315	0.140926	−0.169908	0.014454	0.016519
33	0.55	0.593127	0.132815	−0.175942	0.013783	0.016845
34		0.623526	0.124497	−0.181356	0.013064	0.017084
35		0.653381	0.116042	−0.186089	0.012307	0.017230
36	0.60	0.682560	0.107520	−0.190080	0.011520	0.017280
37		0.710940	0.098997	−0.193271	0.010710	0.017230
38		0.738401	0.090541	−0.195609	0.009887	0.017077
39	0.65	0.764831	0.082213	−0.197043	0.009057	0.016821
40		0.790124	0.074014	−0.197531	0.008230	0.016461
41		0.814183	0.066182	−0.197031	0.007414	0.015998
42	0.70	0.836920	0.058590	−0.195510	0.006615	0.015435
43		0.858258	0.051348	−0.192939	0.005841	0.014775
44		0.878131	0.044500	−0.189298	0.005099	0.014022
45	0.75	0.896484	0.038086	−0.184570	0.004395	0.013184
46		0.913276	0.032140	−0.178750	0.003733	0.012267
47		0.928479	0.026691	−0.171837	0.003121	0.011282
48	0.80	0.942080	0.021760	−0.163840	0.002560	0.010240
49		0.954083	0.017362	−0.154777	0.002055	0.009153
50		0.964506	0.013503	−0.144672	0.001607	0.008038
51	0.85	0.973388	0.010184	−0.133572	0.001219	0.006909
52		0.980784	0.007396	−0.121513	0.000890	0.005786
53		0.986770	0.005120	−0.108556	0.000620	0.004691
54	0.90	0.991440	0.003330	−0.094770	0.000405	0.003645
55		0.994912	0.001989	−0.080235	0.000243	0.002674
56		0.997325	0.001051	−0.065043	0.000129	0.001807
57	0.95	0.998842	0.000457	−0.049299	0.000056	0.001072
58		0.999648	0.000140	−0.033121	0.000017	0.000502
59		0.999955	0.000018	−0.016639	0.000002	0.000132
60	1.00	1.000000	0	0	0	0

Table 2—Velocity Factors

Pt.	x	V_1	V_2	V_3	V_4	V_5
0	0	0	1.00000	0	0	0
3	0.05	0.06769	0.95891	−0.02659	0.03948	0.00321
6	0.10	0.24300	0.85050	−0.09350	0.06075	0.01125
9	0.15	0.48769	0.69541	−0.18309	0.06773	0.02152
12	0.20	0.76800	0.51200	−0.28000	0.06400	0.03200
15	0.25	1.05469	0.31641	−0.37109	0.05273	0.04102
18	0.30	1.32300	0.12250	−0.44550	0.03675	0.04725
21	0.35	1.55269	−0.05809	−0.49459	0.01848	0.04977
24	0.40	1.72800	−0.21600	−0.51200	0.00000	0.04800
27	0.45	1.83769	−0.34409	−0.49359	−0.01702	0.04177
30	0.50	1.87500	−0.43750	−0.43750	−0.03125	0.03125
33	0.55	1.83769	−0.49359	−0.34409	−0.04177	0.01702
36	0.60	1.72800	−0.51200	−0.21600	−0.04800	0.00000
39	0.65	1.55269	−0.49459	−0.05809	−0.04977	−0.01848
42	0.70	1.32300	−0.44550	0.12250	−0.04725	−0.03675
45	0.75	1.05469	−0.37109	0.31641	−0.04102	−0.05273
48	0.80	0.76800	−0.28000	0.51200	−0.03200	−0.06400
51	0.85	0.48769	−0.18309	0.69541	−0.02152	−0.06773
54	0.90	0.24300	−0.09350	0.85050	−0.01125	−0.06075
57	0.95	0.06769	−0.02659	0.95891	−0.00321	−0.03948
60	1.00	0	0	1.00000	0	0

Table 3—Acceleration Factors

Point	x	A_1	A_2	A_3	A_4	A_5
0	0	0	0	0	1.00	0.00
6	0.10	4.32	−2.70	−1.62	0.27	0.19
12	0.20	5.76	−3.84	−1.92	−0.16	0.20
18	0.30	5.04	−3.78	−1.26	−0.35	0.09
24	0.40	2.88	−2.88	0	−0.36	−0.08
30	0.50	0	−1.50	1.50	−0.25	−0.25
36	0.60	−2.88	0	2.88	−0.08	−0.36
42	0.70	−5.04	1.26	3.78	0.09	−0.35
48	0.80	−5.76	1.92	3.84	0.20	−0.16
54	0.90	−4.32	1.62	2.70	0.19	0.27
60	1.00	0	0	0	0	1.00

Table 4—Pulse Factors

Point	x	P_1	P_2	P_3	P_4	P_5
0	0	60.00	−36.00	−24.00	−9.00	3.00
6	0.10	27.60	−18.60	−9.00	−5.70	0.90
12	0.20	2.40	−4.80	2.40	−3.00	−0.60
18	0.30	−15.60	5.40	10.20	−0.90	−1.50
24	0.40	−26.40	12.00	14.40	0.60	−1.80
30	0.50	−30.00	15.00	15.00	1.50	−1.50
36	0.60	−26.40	14.40	12.00	1.80	−0.60
42	0.70	−15.60	10.20	5.40	1.50	0.90
48	0.80	2.40	2.40	−4.80	0.60	3.00
54	0.90	27.60	−9.00	−18.60	−0.90	5.70
60	1.00	60.00	−24.00	−36.00	−3.00	9.00

Design Examples

Dwell-Rise-Dwell Cam—Fixed Intermediate Displacement: Cam action is shown schematically by the displacement diagram in Fig. 2*a*. Find the optimum displacement equations when $h_B = 0.64$ in.; $h_C = 0.36$ in.; $t = 0.04$ sec; $nt = 0.03$ sec; velocity at point *A*, $v_A = 0$; velocity at point *C*, $v_C = 0$; acceleration at point *A*, $a_A = 0$; and acceleration at point *C*, $a_C = 0$.

To assure smoothness and continuity of the derivative curves, final velocity v_B, acceleration a_B, and pulse p_B of curve *AB* should equal the initial velocity, acceleration, and pulse of curve *BC*.

The first step is to equalize the pulse. From Table 4 and Equation 5, the pulse at *B* for curve *AB* is:

$$p_B = 60\frac{h_B}{t^3} - 36\frac{v_B}{t^2} + 9\frac{a_B}{t}$$

$$= 60(10)^4 - 2.25(10)^4 v_B + 2.25(10)^2 a_B \qquad (6)$$

Similarly, for curve *BC*,

$$p_B = 60\frac{h_C}{(nt)^3} - 36\frac{v_B}{(nt)^2} - 9\frac{a_B}{nt}$$

$$= 80(10)^4 - 4(10)^4 v_B - 3(10)^2 a_B \qquad (7)$$

Subtracting Equation 6 from Equation 7,

$$0 = 20(10)^4 - 1.75(10)^4 v_B - 5.25(10)^2 a_B$$

Solving for each variable,

$$a_B = 381 - 33.3\, v_B \qquad (8)$$

$$v_B = 11.43 - 0.03\, a_B \qquad (9)$$

If any value is assumed for a_B and the corresponding value of v_B is determined from Equation 9 and substituted into Equation 4, point by point accelerations can be calculated and plotted. Successive point plots for various values of a_B will eventually produce an optimum curve. However, after two trials a suitable curve can be obtained by inspection.

Inspection Method: For the first trial, let $a_B = -600$. From Equation 9, $v_B = 11.43 - 0.03\,(-600) = 29.43$.

From Equation 4 for curve AB,

$$\frac{d^2y}{dt^2} = A_1\frac{h_B}{t^2} + A_3\frac{v_B}{t} + A_5\,a_B$$

$$= 400\,A_1 + 736\,A_3 - 600\,A_5 \tag{10}$$

Similarly, for curve BC,

$$\frac{d^2y}{dt^2} = A_1\frac{h_C}{(nt)^2} + A_2\frac{v_B}{nt} + A_4\,a_B$$

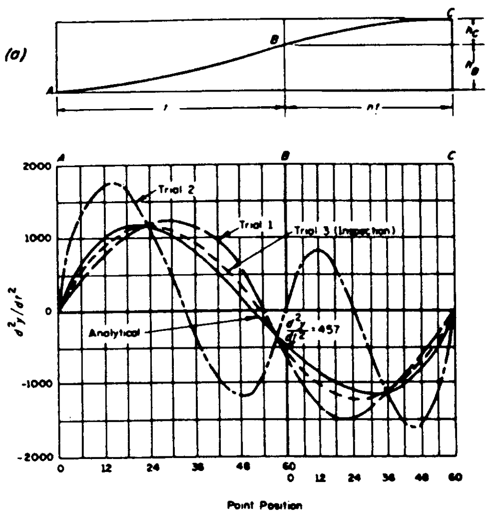
$$= 400\,A_1 + 981\,A_2 - 600\,A_4 \tag{11}$$

For the second trial, let $a_B = 0$. From Equation 9, $v_B = 11.43$. Then, from Equation 4 for curve AB,

$$\frac{d^2y}{dt^2} = 400\,A_1 + 286\,A_3 \tag{12}$$

and for curve BC,

$$\frac{d^2y}{dt^2} = 400\,A_1 + 381\,A_2 \tag{13}$$



(b)

Trial Calculations

First Trial

Point	x	Curve AB $400A_1$	$736A_3$	$-600A_5$	d^2y/dt^2	Curve BC $400A_1$	$981A_2$	$-600A_4$	d^2y/dt^2
0	0	0	0	0	0	0	0	−600	−600
6	0.10	1728	−1192	−114	422	1728	−2649	−162	−1083
12	0.20	2304	−1413	−120	771	2304	−3767	96	−1367
18	0.30	2016	−927	−84	1005	2016	−3708	210	−1482
24	0.40	1152	0	48	1200	1152	−2825	216	−1457
30	0.50	0	1104	150	1254	0	−1471	150	−1321
36	0.60	−1152	2120	216	1184	−1152	0	48	−1104
42	0.70	−2016	2782	210	976	−2016	1236	−54	−834
48	0.80	−2304	2826	96	618	−2304	1884	−120	−540
54	0.90	−1728	1987	−162	97	−1728	1589	−114	−253
60	1.00	0	0	−600	−600	0	0	0	0

Second Trial

Point	x	Curve AB $400A_1$	$286A_3$	d^2y/dt^2	Curve BC $400A_1$	$381A_2$	d^2y/dt^2
0	0	0	0	0	0	0	0
6	0.10	1728	−463	1265	1728	−1029	699
12	0.20	2304	−849	1755	2304	−1463	841
18	0.30	2016	−360	1656	2016	−1440	576
24	0.40	1152	0	1152	1152	−1097	55
30	0.50	0	429	429	0	−571	−571
36	0.60	−1152	823	−329	−1152	0	−1152
42	0.70	−2016	1080	−936	−2016	480	−1536
48	0.80	−2304	1097	−1207	−2304	732	−1572
54	0.90	−1728	772	−956	−1728	617	−1111
60	1.00	0	0	0	0	0	0

Fig. 2—Dwell-rise-dwell cam with intermediate displacement: *a*, displacement diagram; *b*, acceleration diagram and trial calculations.

Discrete values of d^2y/dt^2 as calculated from Equations 10 through 13, using Table 3 to evaluate the A coefficients, are given and plotted in Fig. 2. Slide rule calculations are usually of sufficient accuracy at this stage.

Inspection of the trial curves, Fig. 2*b*, reveals certain characteristics. It can be proved that the three intersections are common to all curves with the specified parameters. Therefore, these intersections establish the minimum peak values of positive and negative accelerations. By visual interpolation, a curve which will approach these minimum accelerations can be sketched. Trial curve 3 (dotted line) closely meets the requirements. In this curve $a_B = -500$, which would become the basis for the final curve. From Equation 9, when $a_B = -500$, $v_B = 26.43$.

From Equation 4, the acceleration equations are:

1. Curve AB,

$$\frac{d^2y}{dt^2} = 400\,A_1 + 661\,A_3 - 500\,A_5 \tag{14}$$

2. Curve BC,

$$\frac{d^2y}{dt^2} = 400\,A_1 + 881\,A_2 - 500\,A_4 \tag{15}$$

From Equation 2, the displacement equations are:

1. Curve AB,

$$y = 0.64\,H_1 + 1.06\,H_3 - 0.80\,H_5 \tag{16}$$

2. Curve BC,

$$y = h_B + 0.36\,H_1 + 0.79\,H_2 - 0.45\,H_4$$

$$= 0.64 + 0.36\,H_1 + 0.79\,H_2 - 0.45\,H_4 \tag{17}$$

Analytical Method: In addition to the trial method, which is quick and efficient, a purely analytical procedure can be used to give a direct solution.

In Fig. 2*b*, trial curve 3 slopes upward to the right at terminals A and C, signifying that the pulse at these points is positive. The minimum accelerations possible are at or near points AB-24 and BC-36. This condition is not only true for this curve, but for any curve with fixed terminal conditions where the intermediate displacements and the times of the events are roughly proportional; that is, the larger displacement occurs in the greater time.

If minimum accelerations are to occur at these points, the slope of the acceleration curve (pulse) must be zero. Also, velocity must be positive at all points. From Equation 5 and Table 4 the pulse at terminal A is:

$$p_A = 60\frac{h_B}{t^3} - 24\frac{v_B}{t^2} + 3\frac{a_B}{t}$$

Substituting the known values, eliminating a_B (Equation 8), and simplifying,

$$p_A = 175(10)^2(35.9 - v_B) \tag{18}$$

Thus, p_A will not be negative if $v_B \leqq 35.9$.

Similarly, the pulse at terminal C is:

$$p_C = 60\frac{h_C}{(nt)^3} - 24\frac{v_B}{(nt)^2} - 3\frac{a_B}{(nt)}$$

$$= 233(10)^2(32.7 - v_B) \qquad (19)$$

Thus, p_C will not be negative when $v_B \leqq 32.7$. Based on Equations 18 and 19, v_B cannot be greater than 32.7.

Let the pulse at point AB-24 equal zero. From Equation 5,

$$0 = -26.4\frac{h_B}{t^3} + 14.4\frac{v_B}{t^2} - 1.8\frac{a_B}{t}$$

Substituting as before and solving give $v_B = 26.8$, which is compatible with the limit condition. Let the pulse at point BC-36 equal zero. Then,

$$0 = -26.4\frac{h_C}{(nt)^3} + 14.4\frac{v_B}{(nt)^2} + 1.8\frac{a_B}{nt}$$

and $v_B = 23.5$, which is also compatible with the limit condition. The logical move now appears to be to take the average of the two computed v_B values. Thus,

$$v_B = \frac{26.8 + 23.5}{2} = 25.15$$

and, from Equation 8,

$$a_B = 381 - 33.3(25.15) = -457$$

This result compares favorably with the value of -500 obtained by inspection.

The acceleration equations are:

1. Curve AB,

$$\frac{d^2y}{dt^2} = 400\,A_1 + 629\,A_3 - 457\,A_5 \qquad (20)$$

2. Curve BC,

$$\frac{d^2y}{dt^2} = 400\,A_1 + 838\,A_2 - 457\,A_4 \qquad (21)$$

The displacement equations are:

1. Curve AB,

$$y = 0.64\,H_1 + 1.006\,H_3 - 0.73\,H_5 \qquad (22)$$

2. Curve BC,

$$y = 0.64 + 0.36\,H_1 + 0.755\,H_2 - 0.41\,H_4 \qquad (23)$$

The acceleration curve (analytical) is shown in Fig. 2b.

Dwell-Rise Cam—Fixed Intermediate Displacement and Fixed Terminal Velocity: Displacement diagram is shown in Fig. 3a. Design conditions are: $h_B = 0.250$ in., $h_C = 0.630$ in., $t = 0.02$ sec; $nt = 0.03$ sec, $v_A = 0$, $v_C = 6.0$, $a_A = 0$, and $a_C = 0$. As in the previous example, first equalize the pulse at B. From Table 4 and Equation 5, for curve AB,

$$p_B = 187.5(10)^4 - 9(10)^4 v_B + 4.5(10)^2 a_B \qquad (24)$$

and for curve BC

$$p_B = 124(10)^4 - 4(10)^4 v_B - 3(10)^2 a_B \qquad (25)$$

Subtracting Equation 24 from Equation 25, and solving for each unknown,

$$a_B = -847 + 66.7\,v_B \qquad (26)$$

$$v_B = 12.7 + 0.015\,a_B \qquad (27)$$

From Table 4 and Equations 5 and 26, the pulse at A is:

$$p_A = 5(10)^4(35 - v_B) \qquad (28)$$

Thus, p_A will not be negative when $v_B \leqq 35$. Similarly, the pulse at C is:

$$p_C = 3.33(10)^4(37.3 - v_B) \qquad (29)$$

Limiting condition is $v_B \leqq 35$. Therefore v_B cannot exceed 35.

Let the pulse at point AB-24 equal zero. From Table 4 and Equations 5 and 26, $v_B = 25$, which is compatible with the limit condition.

Let the pulse at point BC-36 equal zero. Solving as before gives $v_B = 29.3$, which is also compatible with the limit condition. Therefore, let $v_B = (25 + 29.3)/2 = 27.15$. From Equation 26, $a_B = 964$.

The acceleration equations are:

1. Curve AB,

$$\frac{d^2y}{dt^2} = 625\,A_1 + 1357.5\,A_3 + 964\,A_5 \qquad (30)$$

2. Curve BC,

$$\frac{d^2y}{dt^2} = 700\,A_1 + 905\,A_2 + 200\,A_3 + 964\,A_4 \qquad (31)$$

The displacement equations are

1. Curve AB,

$$y = 0.25\,H_1 + 0.543\,H_3 + 0.386\,H_5 \qquad (32)$$

2. Curve BC,

$$y = 0.25 + 0.63\,H_1 + 0.815\,H_2 + 0.18\,H_3 + 0.868\,H_4 \qquad (33)$$

The acceleration curve is shown in Fig. 3b.

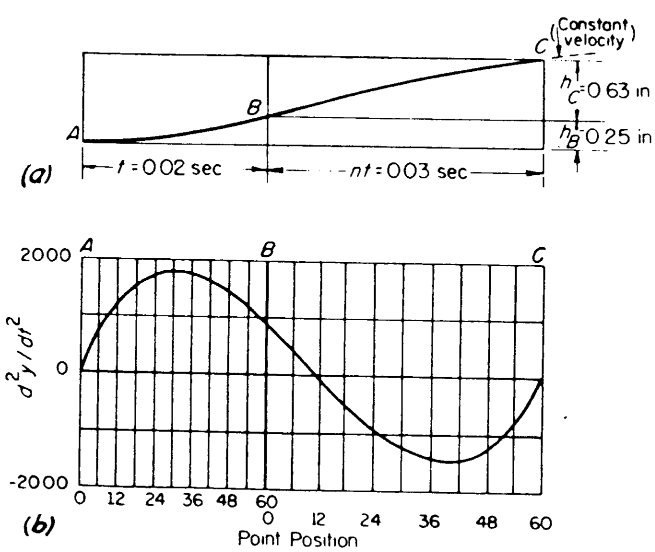


Fig. 3—Dwell-rise cam with intermediate displacement and fixed terminal velocity: *a*, displacement diagram; *b*, acceleration diagram.

Dwell-Rise-Dwell Cam—Fixed Intermediate Displacement with Inverse Time Relationships: In the previous examples, displacements h_B and h_C have occurred in relatively proportional times. When the smaller displacement takes place in the greater time, the procedure becomes slightly more complex. The optimum acceleration curve may not be immediately apparent by the trial and inspection method and, even with the analytical method, some judgment is required to determine the best solution.

The difficulty arises because the minimum peak negative acceleration may not be near point BC-36, as in the previous examples. It may occur between points AB-54 and BC-6 and, for practical purposes, may be assumed to be at point B.

A procedure for checking this cam action is demonstrated here. Consider the displacement diagram shown in Fig. 4a. Design conditions are: h_B = 0.64 in., h_C = 0.36 in., t = 0.03 sec, nt = 0.04 sec, $v_A = 0$, $v_C = 0$, $a_A = 0$, and $a_C = 0$. Using the same procedure as in the previous examples,

$$a_B = -2066 + 33.3\,v_B \tag{34}$$

$$v_B = 62 + 0.030\,a_B \tag{35}$$

Pulse p_A will not be negative when $v_B \leqq 52$, and p_C will not be negative, when $v_B \leqq 28.1$. Therefore, v_B must be between zero and 28.1.

The position of the minimum possible peak negative acceleration can be determined by the following procedure. From Equation 4 and Table 3, the acceleration at point BC-36 is:

$$\frac{d^2y}{dt^2} = -648 - 0.08\,a_B \tag{36}$$

Let $v_B = 0$. Then, from Equation 34, $a_B = -2066$, and from Equation 36, $a_{BC\text{-}36} = -483$. Similarly, when $v_B = 28.1$, $a_B = -1130$ and $a_{BC\text{-}36} = -558$.

Thus, for all possible values of v_B, the acceleration at BC-36 is less numerically than the acceleration at B. Therefore, the minimum peak negative acceleration will occur near B.

Let the pulse at point B equal zero. Solving as before, $v_B = 26.8$.

Let the pulse at point AB-24 equal zero. Then $v_B = 35.8$.

The average of these two values, $v_B = 31.3$, is not compatible with the limit condition $v_B \leqq 28.1$. The value which is the nearest to 31.3, but does not violate the condition of non-negative pulse at the terminals is $v_B = 28.1$. From Equation 34, this value gives $a_B = -1130$.

The acceleration equations are:

1. Curve AB,

$$\frac{d^2y}{dt^2} = 711\,A_1 + 937\,A_3 - 1130\,A_5 \tag{37}$$

2. Curve BC,

$$\frac{d^2y}{dt^2} = 225\,A_1 + 703\,A_2 - 1130\,A_4 \tag{38}$$

The displacement equations are:

1. Curve AB,

$$y = 0.64\,H_1 + 0.843\,H_3 - 1.017\,H_5 \tag{39}$$

2. Curve BC,

$$y = 0.64 + 0.36\,H_1 + 1.124\,H_2 - 1.808\,H_4 \tag{40}$$

The acceleration curve is shown in Fig. 4b.

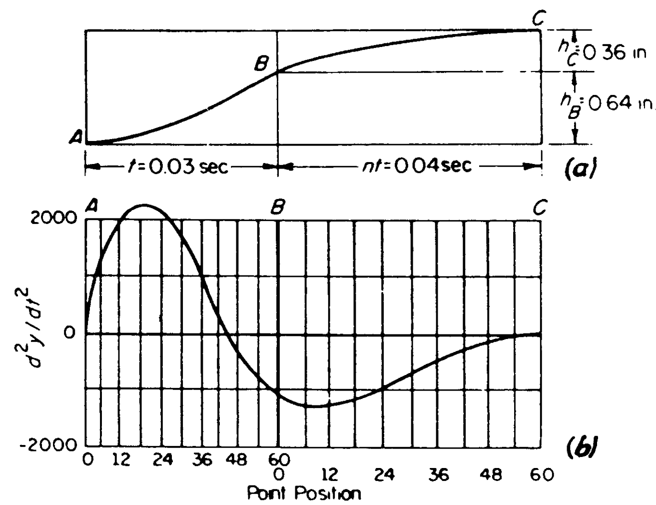


Fig. 4—Dwell-rise-dwell cam with intermediate displacement and inverse time relationships: ***a*, displacement diagram; *b*, acceleration diagram.**

Dwell-Rise-Return-Cams — Basic Considerations: None of the basic curves satisfactorily fulfill the requirements of dwell-rise-return cam action. The cycloidal curve has an unnecessary return to zero acceleration at the maximum rise point. The harmonic curve has infinite pulse at the start of the rise. Combinations of the basic curves, the double-harmonic curve, and certain polynomials have been developed to eliminate these objectionable characteristics. However, when stringent intermediate conditions are introduced, the same difficulties arise as in the D-R-D curve. Basically, the same procedure as described before can be used.

Dwell-Rise-Return Cam—Fixed Intermediate Displacement with Symmetrical Rise and Return: Displacement diagram is shown in Fig. 5a. Design conditions are: h_B = 0.40 in., h_C = 0.60 in., t = 0.02 sec, nt = 0.02 sec, $v_A = 0$, $a_A = 0$, $v_C = 0$, $p_C = 0$. Acceleration a_C must be determined. Note that the pulse at point C is zero to insure tangency of the rise and return acceleration curves at this point.

For curve AB, from Equation 5 and Table 4,

$$p_B = 300(10)^4 - 9(10)^4\,v_B + 4.5(10)^2\,a_B \tag{41}$$

Also, for curve BC,

$$p_B = 450(10)^4 - 9(10)^4\,v_B - 4.5(10)^2\,a_B + 1.5(10)^2\,a_C \tag{42}$$

and

$$p_C = 450(10)^4 - 6(10)^4\,v_B - 1.5(10)^2\,a_B + 4.5(10)^2\,a_C = 0 \tag{43}$$

If Equation 41 is subtracted from Equation 42, simultaneous solution of result and Equation 43 gives

$$a_B = 23.5\, v_B \quad (44)$$

$$a_C = 141.2(-70.8 + v_B) \quad (45)$$

Since a_C must be negative and v_B positive, $v_B < 70.8$. From Table 4 and Equations 5 and 44,

$$p_A = 565(10)^2(53.1 - v_B) \quad (46)$$

Thus, p_A will not be negative if $v_B \leqq 53.1$. Let the pulse at point AB-24 equal zero. From Equation 5 then, $v_B = 39$. Similarly, if the pulse at point BC-36 equals zero, $v_B = 49.6$. Let $v_B = (39 + 49.6)/2 = 44.3$, which is compatible with Equation 46. Then from Equations 44 and 45, $a_B = 1041$ and $a_C = -3742$.

The acceleration equations are:

1. Curve AB,

$$\frac{d^2y}{dt^2} = 1000\, A_1 + 2215\, A_3 + 1041\, A_5 \quad (47)$$

2. Curve BC,

$$\frac{d^2y}{dt^2} = 1500\, A_1 + 2215\, A_3 + 1041\, A_4 - 3742\, A_5 \quad (48)$$

The displacement equations are:

1. Curve AB,

$$y = 0.40\, H_1 + 0.886\, H_3 + 0.416\, H_5 \quad (49)$$

2. Curve BC,

$$y = 0.40 + 0.60\, H_1 + 0.886\, H_2 + 0.416\, H_4 + 1.497\, H_5 \quad (50)$$

The acceleration curve is shown in Fig. 5*b*.

The return is, of course, a mirror image of the rise. Note that if the average of the velocities at points AB-24 and BC-36 had exceeded the velocity limit established by Equations 45 and 46, the choice would be the lower value from these equations.

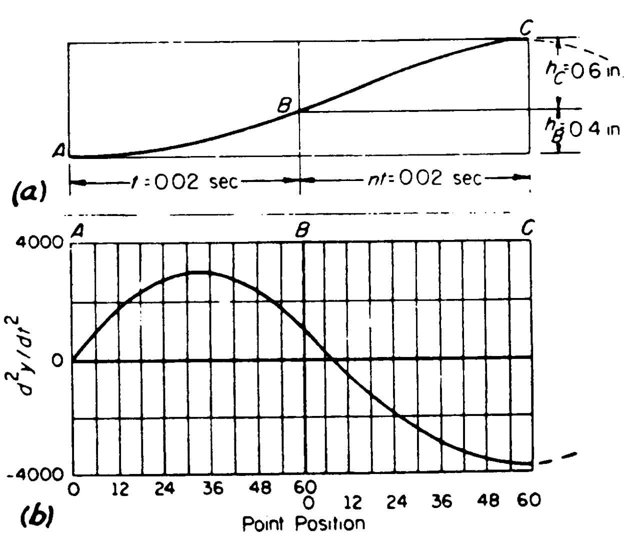


Fig. 5—Dwell-rise-return cam with intermediate displacement and symmetrical rise and return: *a*, displacement diagram; *b*, acceleration diagram.

Dwell-Rise-Return-Cam—Unsymmetrical Periods of Rise and Return: When the function of a cam requires that the rise and return occur in unequal periods of time, the designer is faced with the problems of matching accelerations at the maximum rise point to prevent discontinuity of the acceleration curve. As a solution, Neklutin[2,3] has developed systems of modified trapezoids and modified sine curves, Schmidt[4] has proposed combinations of cycloidal and harmonic curves, and Rothbart[5] has recommended, but not enlarged on, the use of polynomial curves. With the methods demonstrated here, a set of general equations which will meet the requirements of this type of curve can be developed.

A typical displacement diagram is shown in Fig. 6*a* where L = total displacement, $2t$ = longer time period, and $2nt$ = shorter time period. Intermediate displacements, h_B, h_C, h_D, and h_E are to be determined, as well as the matching acceleration, a_C, at point C. Also, $v_A = 0$, $v_E = 0$, $a_A = 0$, $a_E = 0$, and $p_C = 0$.

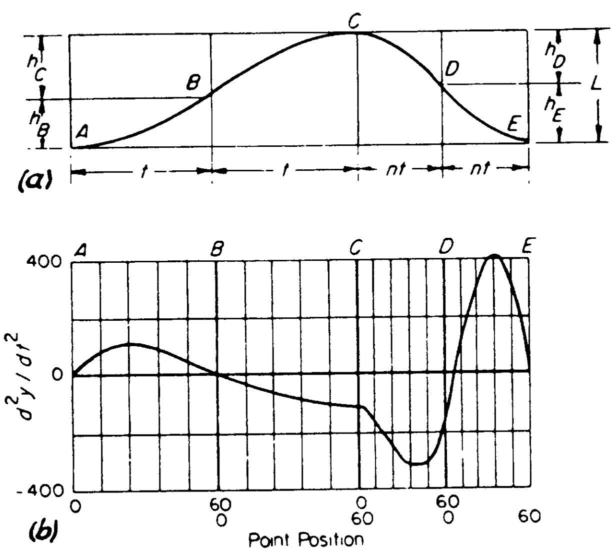


Fig. 6—Dwell-rise-return cam with unsymmetrical periods of rise and return: *a*, displacement diagram; *b*, acceleration diagram.

For curve AB, solving as before,

$$p_B = 60\frac{h_B}{t^3} - 36\frac{v_B}{t^2} + 9\frac{a_B}{t} \quad (51)$$

and for curve BC,

$$p_B = 60\frac{(L-h_B)}{t^3} - 36\frac{v_B}{t^2} - 9\frac{a_B}{t} + 3\frac{a_C}{t} \quad (52)$$

Subtracting Equation 51 from Equation 52,

$$0 = 60\frac{L}{t^3} - 120\frac{h_B}{t^2} - 18\frac{a_B}{t} + 3\frac{a_C}{t} \quad (53)$$

from which,

$$h_B = 0.500\, L - 0.150\, a_B t^2 + 0.025\, a_C t^2 \quad (54)$$

and

$$h_C = 0.500\, L + 0.150\, a_B t^2 - 0.025\, a_C t^2 \quad (55)$$

From Equation 5,

$$p_C = 60 \frac{L - h_B}{t^3} - 24 \frac{v_B}{t^2} - 3 \frac{a_B}{t} + 9 \frac{a_C}{t} = 0 \qquad (56)$$

Solving Equations 53 and 56 simultaneously,

$$v_B = 1.250 \frac{L}{t} + 0.250\, a_B t + 0.3125\, a_C t \qquad (57)$$

To establish a relationship between a_B and a_C, assume that the maximum peak positive acceleration will occur at point *AB*-24. Thus the pulse at this point will be zero:

$$0 = -26.4 \frac{h_B}{t^3} + 14.4 \frac{v_B}{t^2} - 1.80 \frac{a_B}{t} \qquad (58)$$

Substituting Equations 54 and 57 into Equation 58 and solving,

$$a_C = -1.250 \frac{L}{t^2} - 1.50\, a_B \qquad (59)$$

Advantage can be taken of the desirability of having the maximum positive and negative accelerations the same numerical value. Therefore, let the negative value of the acceleration at point *AB*-24 equal the acceleration at point *C*. From Table 3 and Equation 4 then,

$$a_C = -\left(2.88 \frac{h_B}{t^2} - 0.08\, a_B \right) \qquad (60)$$

Substituting Equation 54 into Equation 60 and solving,

$$a_C = -1.343 \frac{L}{t^2} + 0.478\, a_B \qquad (61)$$

Combining Equations 59 and 61,

$$a_B = 0.047 \frac{L}{t^2} \qquad (62)$$

and from Equation 59 or 61,

$$a_C = -1.321 \frac{L}{t^2} \qquad (63)$$

From Equations 54, 55, and 57,

$$h_B = 0.460\, L \qquad (64)$$

$$h_C = 0.540\, L \qquad (65)$$

$$v_B = 0.849 \frac{L}{t} \qquad (66)$$

By similar methods, assuming that the pulse at point *DE*-36 is zero and recognizing that displacements and velocities are negative for the return, and the value of a_C is fixed, these equations can be derived for the return curve, *CDE*:

$$h_D = -L(0.375 + 0.165\, n^2) \qquad (67)$$

$$h_E = -L(0.625 - 0.165\, n^2) \qquad (68)$$

$$v_D = -L \frac{\dfrac{1.042}{n} - 0.193\, n}{t} \qquad (69)$$

$$a_D = -L \frac{\dfrac{0.833}{n^2} - 0.880}{t^2} \qquad (70)$$

Note that if $n = 1$, these equations are identical with those for curve *ABC*, except for the signs of displacement and velocity.

Use of these equations is best demonstrated with an example. In the displacement diagram, Fig. 6*a*, let $L = 2.00$ in., $t = 0.15$ sec, $nt = 0.09$ sec, and $n = 0.60$. Then, from Equations 64, 65, and 66, $h_B = 0.920$ in., $h_C = 1.080$ in., and $v_B = 11.32$ ips. From Equations 62 and 63, $a_B = 4.20$ in./sec² and $a_C = -117.4$ in./sec². Finally, from Equations 67 through 70, $h_D = -0.869$ in., $h_E = -1.131$ in., $v_D = -21.6$ ips, and $a_D = -127.5$ in./sec².

The acceleration equations are:

1. Curve *AB*,

$$\frac{d^2y}{dt^2} = 40.9\, A_1 + 75.5\, A_3 + 4.20\, A_5 \qquad (71)$$

2. Curve *BC*,

$$\frac{d^2y}{dt^2} = 48\, A_1 + 75.5\, A_3 + 4.20\, A_4 - 117.4\, A_5 \qquad (72)$$

3. Curve *CD*,

$$\frac{d^2y}{dt^2} = -107.3\, A_1 - 240\, A_3 - 117.4\, A_4 - 127.5\, A_5 \qquad (73)$$

4. Curve *DE*,

$$\frac{d^2y}{dt^2} = -139.6\, A_1 - 240\, A_2 - 127.5\, A_4 \qquad (74)$$

The displacement equations are:

1. Curve *AB*,

$$y = 0.920\, H_1 + 1.698\, H_3 + 0.095\, H_5 \qquad (75)$$

2. Curve *BC*,

$$y = 0.920 + 1.08\, H_1 + 1.698\, H_2 + 0.095\, H_4 - 2.642\, H_5 \qquad (76)$$

3. Curve *CD*,

$$y = 2.00 - 0.869\, H_1 - 1.944\, H_3 - 0.951\, H_4 - 1.033\, H_5 \qquad (77)$$

4. Curve *DE*,

$$y = 1.131 - 1.131\, H_1 - 1.944\, H_2 - 1.033\, H_4 \qquad (78)$$

The acceleration curve is shown in Fig. 6*b*.

Special Cam Problems

CAM IN LIMITED SPACE

Problem Statement Sometimes conditions, such as available space or clearance of adjacent parts, make it necessary to design a cam of limited major radius. If angular and linear displacements must be held to fixed limits, the largest possible cam diameter may result in a maximum pressure angle greater than considered to be satisfactory. If specifications will not allow an increase in angular displacement, or a decrease in linear displacement, a combination curve will usually provide a solution.

Curve Comparison Figure AD-1 (a) shows typical displacement diagrams of a D-R-D cam, in which the angular displacement (β), the linear displacement (h), and the pitch radius (R_P), are fixed. The dotted line represents a cycloidal motion resulting in an excessive maximum pressure angle. The full line shows the combination of half-cycloidal and constant velocity curves, which reduce the pressure angle.

Figures AD-1 (b) and (c) show a comparison of the velocity and acceleration diagrams of the basic and combination curves. Note that maximum accelerations of the latter are the greater. This is not always objectionable, as the critical radius of curvature of the combination curve may be better, resulting in reduced contact stress.

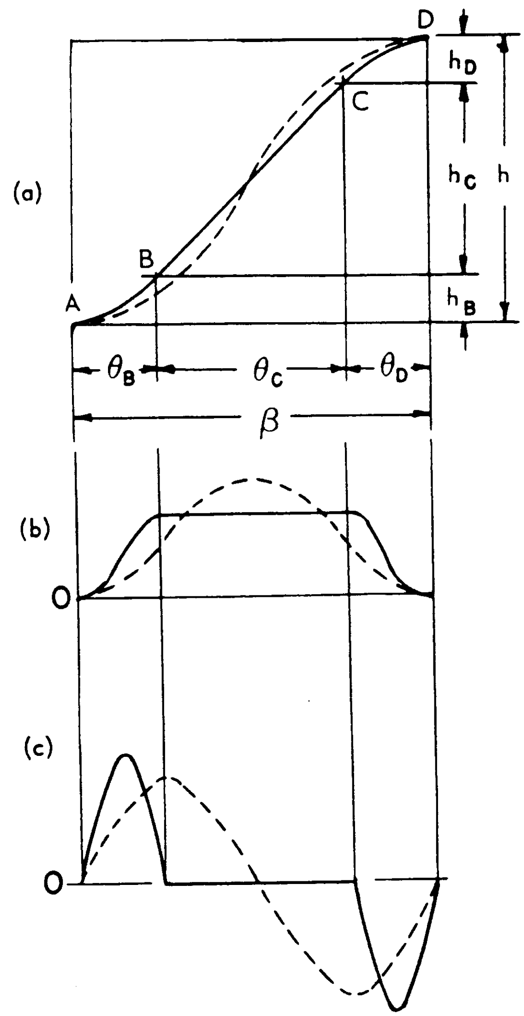


FIG. AD-1

(a) Displacement (b) Velocity (c) Acceleration

Equations. The intermediate angular and linear displacements of the combination curve may be found from the following equations, in which γ represents the reduced pressure angle.

$$h_B = h_D = \frac{R_P \beta \tan \gamma}{114.6} - 0.5\,h \qquad (1)$$

$$h_C = 2\,h - \frac{R_P \beta \tan \gamma}{57.3} \qquad (2)$$

$$\theta_B = \theta_D = \beta - \frac{57.3\,h}{R_P \tan \gamma} \qquad (3)$$

$$\theta_C = \frac{114.6\,h}{R_P \tan \gamma} - \beta \qquad (4)$$

Generally, the maximum pressure angle should not exceed 30°. Therefore, if this angle is chosen as 29.81°, the tangent is 0.573, and the equations are simplified to:

$$h_B = h_D = 0.005\,R_P \beta - 0.5\,h \qquad (5)$$

$$h_C = 2h - 0.01\,R_P \beta \qquad (6)$$

$$\theta_B = \theta_D = \beta - h/0.01\,R_P \qquad (7)$$

$$\theta_C = h/0.005\,R_P - \beta \qquad (8)$$

The displacement equations are

1. Curve AB. (Use Table E-5, points 0 to 60.)

$$y = 2\,Kh_B \qquad (9)$$

2. Curve BC. (Use Table E-1, points 0 to 120.)

$$y = h_B + Kh_C \qquad (10)$$

3. Curve CD. (Use Table E-5, points 60 to 120.)

$$y = h_B + h_C + (2\,K - 1)\,h_D \qquad (11)$$

Example. Given R_P = 1.5 in., h = 1.0 in., B = 90°; N = 200RPM.

From Section F, Equation 2, the maximum pressure angle for a full cycloidal curve is

$$\gamma = \tan^{-1}\frac{(2)\,(57.3)\,(1.0)}{1.5\,(90)} = 40.36^\circ$$

For a pressure angle of 29.81°, applying equations 5 through 8

$$h_B = h_D = 0.005\,(1.5)\,(90) - 0.5\,(1.0) = 0.175 \text{ in.}$$

$$h_C = 2\,(1.0) - 0.01\,(1.5)\,(90) = 0.650 \text{ in.}$$

$$\theta_B = \theta_D = 90 - 1.0/0.01\,(1.5) = 23.333^\circ$$

$$\theta_C = 1.0/0.005\,(1.5) - 90 = 43.333^\circ$$

From Equations 9 thru 11, the displacement equations are

1. Curve AB

$$y = 0.35\,K$$

2. Curve BC

$$y = 0.175 + 0.65\,K$$

3. Curve CD

$$y = 0.650 + 0.350K$$

Curve characteristics. Peak velocities and accelerations of the curves are 26.6 in./sec. and 1120 in./sec.² for the full cycloidal, 18 in./sec. and 1450 in./sec.² for the combination curve. The least radii of curvature are 0.825 in. and 1.53 in. respectively.

SHORT STROKE VS. LONG STROKE CAMS

Design Considerations. A basic concept of good cam design is minimum displacement in maximum time. However, conditions arise where it is advantageous to increase the displacement beyond the minimum required, with a moderate increase in the time of the event. A cam problem of this type is demonstrated in the following example.

Design Example. Consider a cam which operates a tool in conjunction with a dial which is driven intermittently by a geneva motion or an indexing cam. Assume that a displacement of one inch for clearance is required before the dial starts to rotate. Also, assume that this displacement is to occur in 60° of cam rotation and that it is to be cycloidal motion. The dotted line of Fig. AD-2 shows the displacement diagram of a cam with this minimum stroke. The full lines show a diagram with increased displacement and time. The problem is to determine the angular displacement (time) required for the greater linear displacement while maintaining the one inch clearance at 60°.

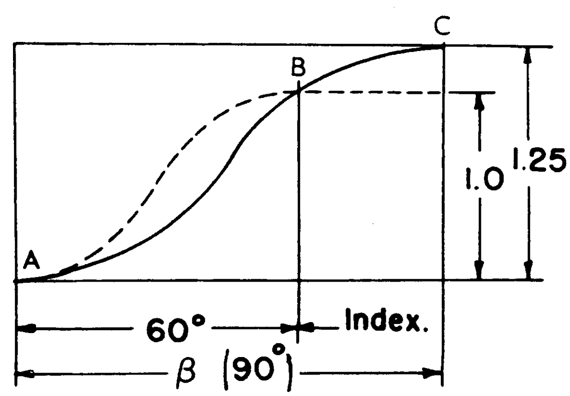


FIG. AD-2

Design Solution: Assume the cam stroke is lengthened to 1¼ inches. The proportion of clearance to full displacement is,

$$K = 1/1.25 = 0.800.$$

From Table E-4 the nearest K factor is 0.80450 at the 80th point of the 120 point table. As this factor is to occur at 60°, the full angular displacement required is

$$\beta = 120\,(60)/80 = 90^\circ.$$

Kinematic Comparison. Comparison of the kinematic characteristics of the two cams shows the advantages of the longer stroke. Assume that the cam rotates at 150 RPM. The peak velocity of the short stroke cam is

$$v = 2\,(1)\,(6 \times 150/60) = 30 \text{ in./sec.}$$

The peak acceleration is,

$$a = 6.28\,(1)\,(6 \times 150/60)^2 = 1413 \text{ in/sec.}^2$$

The peak velocity of the long stroke cam is

$$v = 2\,(1.25)\,(6 \times 150/90) = 25 \text{ in./sec.}$$

The peak acceleration is

$$a = 6.28\,(1.25)\,(6 \times 150/90)^2 = 785 \text{ in./sec.}^2$$

The long stroke cam has lower peak velocities and accelerations. The maximum velocity is reduced 17% and the maximum acceleration, 45%.

Physical Comparison. Assume that the short stroke cam has a 3.312 pitch radius. The major radius is,

$$R_N = 3.312 + 1.0/2 = 3.812 \text{ in.}$$

The maximum pressure angle is

$$\gamma = \tan^{-1} 57.3\,(2)\,(1.0)\,/3.312\,(60) = 30^\circ.$$

Maintaining the same size for the long stroke cam, the pitch radius is

$$R_P = 3.812 - 1.25/2 = 3.187 \text{ in.}$$

The maximum pressure angle is

$$\gamma = \tan^{-1} 57.3\,(2)\,(1.25)\,/3.187\,(90) = 26.54^\circ.$$

Again, the long stroke cam is better, having a lower maximum pressure angle.

Approaching the physical size in a different manner, the pitch radius of the long-stroke cam for approximately a 30° pressure angle is

$$R_P = 200\,(1.25)\,/90 = 2.8 \text{ in.}$$

The major radius, then, is

$$R_N = 2.8 + 1.25/2 = 3.425 \text{ in.}$$

Therefore, the long stroke cam could be about 13/16 inches smaller in diameter and be kinematically satisfactory.

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