Force-Based Roll Centers and an Improved Kinematic Roll Center

Wm. C. Mitchell Wm. C. Mitchell Software



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400 Commonwealth Drive, Warrendale, PA 15096-0001 U.S.A. Tel: (724) 776-4841 Fax: (724) 776-0790 Web: www.sae.org

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ABSTRACT

Roll Centers are an important tuning tool but their importance is often misunderstood. The roll center has too often become a mysterious concept rather than a simple descriptive parameter. Roll Centers may be determined from forces (the Force-Based Roll Center) or the more familiar kinematic method. This paper will explain and reconcile the difference between these roll centers. That explanation will show how the roll center should be used in understanding the behavior of a vehicle.

The most familiar roll centers are for independent front suspensions with two a-arms or wishbones, also known as SLA or Short Long Arm suspensions. For these suspensions this paper will derive a method to construct a more accurate kinematic roll center which explains the differences between the Force-Based Roll Center (FBRC) and the Kinematic Roll Center (KRC).

The common kinematic roll center is based on four links between the upright/spindle/wheel/tire and the sprung mass of the chassis. But there are five links between those parts: the missing link is the steering tie-rod. The steering link accounts for the differences between FBRC and KRC. Proper accounting for the forces on the steering link explains the differences.

The difference between FBRC and KRC would be more significant but the following analysis will show that minimizing bump steer reduces the difference. The practical desire to eliminate bump steer minimizes the theoretical difference between FBRC and KRC.

DEFINITION OF THE ROLL CENTER

SAE defines the roll center as "The point in the transverse vertical plane through any pair of *wheel centers* at which lateral forces may be applied to the

sprung mass without producing *suspension roll*⁷. [1]. This really defines a *height* rather than a point since there is no reference to lateral location. The roll center is often assumed to be on the centerline of the vehicle.

Terry Satchell describes the roll center in Chapter 17 of Race Car Vehicle Dynamics. "The roll center establishes the *force coupling* point between the unsprung and sprung masses. When a car corners, the centrifugal force at the center of gravity is reacted by the tires. The lateral force at the CG can be translated to the roll center if the appropriate force and moment (about the roll center) are shown." [3]

Both of these definitions are based on forces rather than geometry or kinematics.

THE KINEMATIC INSTANT CENTER

Locating a roll center begins with the kinematic instant center.

Satchell continues in Race Car Vehicle Dynamics: "When we connect a line between the ball joint and the control arm bushing and project it across the plane both for the upper and lower control arms they will usually intersect at some point. The intersection is an instantaneous linkage center. If you do the projection in the front view the instant center defines the camber change rate, part of the roll center information, scrub motion, and data needed to determine the steer characteristics." [3]

This is an example of a four-bar linkage consisting of the chassis, the upright/spindle/wheel, an upper link and the lower link. This is a two dimensional object and the upper and lower links are the intersection of the planes containing the a-arms (or wishbones) and the transverse vertical plane through wheel centers.



Figure 1 The Kinematic Instant Center is the intersection of the projected upper and lower a-arms.

THE KINEMATIC ROLL CENTER

The kinematic roll center of a SLA suspension requires the 4-bar linkage theory to be applied twice. The first 4bar linkage determines the kinematic instant center. The upright/spindle/wheel moves as if it were attached to the instant center.

The second 4-bar linkage assumes the sprung mass is attached to the ground by links from each tire to the instant center. If you assume the wheel and tires are pinned to the ground then the sprung mass can only rotate about the kinematic roll center.

Satchell writes "The roll center height is found by projecting a line from the center of the tire-ground contact patch through the front view instant center as shown in (). This is repeated for each side of the car. Where these two lines intersect is the roll center of the sprung mass of the car, relative to the ground. It is not necessarily at the centerline of the car, especially with asymmetric suspension geometry () or once the car assumes a roll angle in a turn." [3]



Figure 2 The Kinematic Roll Center is the intersection of lines from the kinematic instant centers to the tire contact points.

This is such a fundamental application of 4-bar mechanics that one book [2] presents it an exercise to be solved by the student. The exercise is accompanied by the drawing of an asymmetric double A-arm suspension, though the upper A-arm is inclined in the wrong direction.

CRUCIAL ASSUMPTIONS

It is important to remember the assumptions underlying

the kinematic roll center. The kinematic instant center is defined by *wheel centers*. It describes movement of the rigid wheel, which is connected to the rigid chassis with rigid links, and ignores the less rigid tire with a contact point that moves with camber and toe changes.

The second 4-bar application treats the lines from tire contact points to the kinematic instant centers as solid links. It requires the tires be pinned to the track. We know the vehicle track often changes as the sprung mass moves.

The kinematic roll center is a useful summary for symmetric suspensions. The roll center describes the distribution of forces graphically. This promotes understanding. But with asymmetric suspensions the concept has been given more importance than it deserves.

The sprung mass might **want** to rotate about the kinematic roll center, but it **moves** in response to forces and depends upon spring and bar rates [6].

THE FORCE-BASED ROLL CENTER

The force-based roll center is determined by applying a lateral force at the tire contact patch. The tire/wheel/upright is considered a single solid object. The tire force is resisted by the necessary six links: the two upper A-arm links, the two lower A-arm links, the steering tie-rod and a vertical jacking force at the tire contact point. Five of these links transfer forces to the sprung mass of the chassis. The total moment operating on the chassis can be determined by taking the total moment and deducting the jacking force at the tire contact patch.

Philip Morse and John M. Starkey provide a detailed description and physical test in [7]



Figure 3 Lateral force applied at the tire generates a overturning moment.

A GEOMETRIC INTERPERTATION OF THE FORCE-BASED ROLL CENTER

In two dimensions the Force-Based Roll Center has a single upper link, a single lower link and a vertical jacking force. The upper link is the intersection of a vertical transverse plane through the tire contact patch and the plane of the upper control arm. The lower link is the intersection of the same vertical transverse plane and the plane of the lower control arm. These intersection lines often do not go through the ball joint at the outer end of the control arm. This determination is the most complex part of determining a two-dimensional kinematic instant center.



Figure 4 Lateral force produced at the tire contact point is resisted by the upper and lower links and a jacking force.

The forces at the ball joints must be directed along the links. These forces may be translated to the inboard end of the links where they are applied to the sprung mass. The forces may also be translated to their intersection, which by definition is the kinematic instant center.



Figure 5 Lateral forces are transmitted to the kinematic instant center.

At the intersection point the forces can be combined into one resultant. Because this force and the vertical jacking force must equal the initial tire force, the resultant is equal to the lateral input force plus the opposite of the vertical jacking force.

Now consider moments about the tire contact point. Both

the lateral input force and the vertical jacking force operate through this point and thus create no moment. Thus the resultant force can create no moment, which requires the resultant vector to be aimed at the tire contact point. Now we can move the resultant vector along this line to a point under the center of gravity.

This is the Force Application Point.



Figure 6 Lateral forces transmitted to the Force Application Point under the Center of Gravity.

The moment is equivalent to the lateral input force multiplied by the distance between the CG and the force application point, which is also the roll center height. There is also a vertical jacking force acting on the CG that is opposite the jacking force at the tire contact point.

This construction of a 2-dimensional force-based roll center produces the same result as the kinematic version. This is a force-based model which provides useful understanding based on suspension geometry.

THE FORCE-BASED ROLL CENTER IN THREE DIMENSIONS

If we extend the previous model to three dimensions we must add the steering tie-rod. When a lateral force is applied at the tire contact patch, caster trail will produce a force along tie-rod The force along the tie-rod can be moved to the inboard end where it applies to the chassis. Then the force can be translated to a point near the force intersection at the kinematic instant center.



Figure 7 Steering Tie-rod force projected to the kinematic instant center.

This graphic shows how the forces on the steering tie-

rod alter the forces at the force intersection point. This alters the roll center height. But the steering tie-rod is usually aimed at the kinematic instant center, which is equivalent to the force intersection point, to minimize bump steer. This means the tie-rod correction is usually small.

RECONCILING ROLL CENTERS

The difference between the kinematic and force-based roll centers can be reconciled by examining the forces going through the steering tie-rod. The method is based on that used for solid rear-axle suspensions wherein the force is distributed between a Panhard rod (or track rod) located behind the rear axle and an instantaneous center of the truck arms located ahead of the rear axle. (That rustling sound you hear is European designers opening the previously-unread portion of chapter 17 of Milliken and Milliken devoted to solid rear axles.)

"The roll axis determination is made by finding the lateral restraint points. We know the track bar provides lateral restraint so the point where its centerline crosses the centerline of the car is one of them. The other restraint point is determined from the lower control arms." [3]

In this analysis the Panhard rod is located behind the rear axle and the truck arms are located in front. This system is treated as a lever with the fulcrum at the rear axis. This permits the forces to be allocated between the Panhard rod and the truck arms. The roll center height is calculated by apportioning the RC height due to the Panhard rod and the RC height due to the truck arms.

The same method can be used with the front suspensions. The majority of the forces go through the A-arms and the RC height is determined from the kinematic roll center. But some force goes through the steering tie-rod. The amount of this force depends on caster trail and the length of the steering arm. These forces contribute to the overall RC height in proportion to the forces going through the tie-rod and where the tierod is aimed relative to the instant center.



Figure 8 Kinematic Instant Center and Steering Tie-rod projection

THE INSTANT CENTER AXIS

In two dimensions you can construct a kinematic roll center (front view) or a pitch center (side view). In three dimensions you can construct an instantaneous axis of rotation connecting the roll center and pitch center. This is described by Manes and Starkey [8]. This is an elegant concept, but it does not really add any information. If you construct the three-dimensional axis and then reduce it to a front-view instant center and a side-view pitch center, you get the same results as the two-dimensional analysis.

The lateral forces applied at the tire contact patch are distributed between the upper and lower A-arms and the steering tie-rod. This apportionment can be approximated by comparing the caster trail to the sideview distance from the steering/kingpin axis to the steering tie-rod. Visualize it as a lever pivoting about the steering/kingpin axis.

The forces operating through the A-arms are concentrated at the instant center. The forces operating through the steering tie-rod are concentrated at the point where the tie-rod projected is above or below the instant center. If the projection is above, and the forces on the tie-rod are positive, then the enhanced roll center is slightly above the kinematic roll center. If below the roll center is slightly lowered.



FORCE APPLICATION POINT

The Force Application Point is where the line from the tire contact patch to the instant center passes under the center of gravity of the vehicle. This point makes the moment arm perpendicular to the lateral force. Any other moment arm requires a correction to the length of the arm because the forces are applied at an angle.

The Force Application Point is under the CG at the height of the roll center, if you replace "centerline" of the vehicle with "under the CG." The existing definitions using "centerline" are biased toward symmetric cars. Most oval track racing cars are asymmetric. Even road cars can be asymmetric when they have an uneven passenger load.

See Figures 3, 4 and 6.

ESTIMATING TIE-ROD FORCE

The first step is to estimate the force going through the steering tie-rod. This can be done by calculating the torque about the kingpin axis. The A-arm links can not resist this torque because they are aimed at the axis. The input is a lateral force applied at the tire contact patch. This is perpendicular to the jacking force "link". This leaves the force on the steering tie-rod to balance the torque.



Figure 10 Estimating Tie-rod Force by considering moments about the Kingpin axis.

The forces are inversely proportional to the distances to the kingpin axis.



Figure 11 Lateral forces are balanced about the kingpin axis.

Estimating the force on the tie-rod permits an estimated roll center based on geometry alone. The estimate can be improved by using the actual forces on the tie-rod.

One lesson of this analysis is that there is more force on the steering tie-rod than many expect.

REAR STEER EXAMPLE

The rear steer example has the lower A-arm and the steering tie-rod 160 mm above ground. With a flat lower A-arm the instant center is also 160 mm above ground. When we raise or lower the steering tie-rod inboard point the force-based roll center changes but the kinematic roll center height does not change. We can estimate the FBRC using either the actual forces on the tie-rod or the geometric estimate.

Here are the numeric details for the tie-rod at +40 or 200 mm above the ground. The kinematic instant center is at -2406 and 160 mm height. The kinematic roll center on the centerline is 41.339 mm = 160*838.2 / (2406 + 838.2) where the 838.2 mm is the distance from the centerline to the tire contact patch.

The tie-rod projection crosses the instant center axis 33.75 mm above the axis. This represents a torque about the instant center axis. It can be translated to the transverse vertical plane where we place the instant center. This leads to an adjustment of 8.720 = 33.75*838.2 / (2406 + 838.2).

The estimated tie-rod force is -0.2105. Multiplying this force by 8.720 yields 1.836. 41.339 + 1.836 = 43.175 mm which compares to FBRC of 43.111. The difference is 0.064 mm



Figure 12 The Steering Tie-Rod intersects the Instant Center Axis.

The gap is 8.720 mm.



Figure 13 Detail of the intersection between Tie-rod and the Instant Center Axis.

Raising both ends of the steering tie-rod, keeping it flat, does the following:

| Tie-Roo | d FBRC | Force | Geometry | Force | Geo. |
|---------|--------|--------|----------|--------|--------|
| height | height | est. | est. | 8 | est. |
| -40 | 39.135 | 39.135 | 39.063 | -0.184 | -0.190 |
| -30 | 39.592 | 39.592 | 39.534 | -0.187 | -0.193 |
| -20 | 40.059 | 40.059 | 40.016 | -0.189 | -0.195 |
| -10 | 40.537 | 40.537 | 40.510 | -0.191 | -0.198 |
| 0 | 41.027 | 41.027 | 41.016 | -0.193 | -0.200 |
| 10 | 41.529 | 41.529 | 41.535 | -0.196 | -0.203 |
| 20 | 42.043 | 42.043 | 42.068 | -0.198 | -0.205 |
| 30 | 42.570 | 42.570 | 42.614 | -0.201 | -0.208 |
| 40 | 43.111 | 43.111 | 43.175 | -0.203 | -0.211 |
| | | | | | |

Table 1: Rear Steer with flat tie-rod

The estimate using the actual force agrees to 0.001 mm. The estimate based on geometry is within 0.070 mm because the estimated tie-rod force is not exact.

Raising or lowering only the inboard point of the steering tie-rod inclines the tie-rod. An inclined tie-rod creates a much larger gap between the projection and the instant center axis. But the estimated forces are less accurate than when the tie-rod is flat.

| Tie-rod | FBRC | Force | Geometr | У |
|----------|----------|------------|---------|---------|
| inboard | height | estimate | estimat | е |
| -40 | 27.103 | 27.044 | 29.22 | 8 |
| -30 | 30.619 | 30.594 | 32.18 | 8 |
| -20 | 34.111 | 34.104 | 35.14 | 9 |
| -10 | 37.580 | 37.579 | 38.10 | 9 |
| 0 | 41.027 | 41.027 | 41.07 | 0 |
| 10 | 44.451 | 44.452 | 44.03 | 1 |
| 20 | 47.853 | 47.859 | 46.99 | 1 |
| 30 | 51.232 | 51.255 | 49.95 | 2 |
| 40 | 54.590 | 54.645 | 52.91 | 2 |
| Table 2: | Rear ste | eer with i | nclined | tie-rod |

Clearly moving the steering tie-rod does move the forcebased roll center height. The estimates are reasonably accurate with the one using the actual forces the more accurate.

FRONT STEER EXAMPLE

Move the tie-rod to a front steer position. Note that the lower ball joint is directly above the tire contact patch.

| Tie-Roo | 1 FBRC | Force | Geometry | Force | Geo. |
|---------------------------------------|--------|--------|----------|-------|-------|
| height | height | est. | est. | 8 | est. |
| -40 | 43.111 | 43.111 | 43.175 | 0.203 | 0.211 |
| -30 | 42.570 | 42.570 | 42.614 | 0.201 | 0.208 |
| -20 | 42.043 | 42.043 | 42.068 | 0.198 | 0.205 |
| -10 | 41.529 | 41.529 | 41.535 | 0.196 | 0.203 |
| 0 | 41.027 | 41.027 | 41.016 | 0.193 | 0.200 |
| 10 | 40.537 | 40.537 | 40.510 | 0.191 | 0.198 |
| 20 | 40.059 | 40.059 | 40.016 | 0.189 | 0.195 |
| 30 | 39.592 | 39.592 | 39.534 | 0.187 | 0.193 |
| 40 | 39.135 | 39.135 | 39.063 | 0.184 | 0.190 |
| Table 2: Pear stear with flat tie red | | | | | |

Table 3: Rear steer with flat tie-rod.

The estimate using the actual force agrees to 0.001 mm. The estimate based on geometry is within 0.070 mm because the estimated tie-rod force is not exact.

Comparing this with Table1 shows that switching from front steer to rear steer reverses the direction of the force on the tie-rod. This changes the effect of raising or lowering the tie-rod.

BUMP STEER AND THE ROLL CENTER

The FBRC and Kinematic Roll Centers are identical when the tie-rod is aimed at the instant center axis. This is usually a design objective because it minimizes bump steer.

The projection of the steering tie-rod also describes the first-order bump steer. If the tie-rod is not aimed at the KRC then you will have first-order bump steer. Only with larger displacements do the lengths of the a-arms come into play as a second-order effect. Since most designers desire to minimize bump steer, it is rare that the tie-rod will be aimed far from the kinematic instant center axis. Consequently the tie-rod correction will usually be quite small.

ROLL CENTER AND ANTI-DIVE, ANTI-SQUAT

This method also explains the effect of anti-dive and anti-squat on the force-based roll center. In the following example the lower A-arm was rotated to change the side-view swing-arm, and thus the anti-values, without changing the kinematic roll center. The lower ball joint is in the plane of the wheel centers and the arms are symmetric. By raising one end and lowering the other end we maintain the same instant center and kinematic roll center. But the force-based roll center changes because we change the interaction between the tie-rod projection and instant center axis.

All of these cases have the kinematic roll center at 41.339. The steering tie-rod is aimed at the instant center to minimize bump steer. Yet introducing antifactors changes the force-based roll center.

This suggests the steering tie-rod should be aimed at the instant center axis rather than the instant center. The distinction is usually small.

| Tie-rod height | FBRC height | Force est. | Geometi est. | сy |
|-------------------|----------------|---------------|-----------------|---------|
| -15 15 | 40.402 | 40.402 | 40.370 | |
| -10 10 | 40.715 | 40.715 | 40.693 | |
| -5 5 | 41.027 | 41.027 | 41.016 | |
| 0 0 | 41.339 | 41.339 | 41.339 | |
| 5 -5 | 41.651 | 41.651 | 41.662 | |
| 10 -10 | 41.963 | 41.963 | 41.985 | |
| 15 -15 | 42.275 | 42.275 | 42.308 | |
| Table 4: | Anti-Dive | and Ant: | i-Squat | effects |

If we change the example from rear steer to front steer, the same pattern would appear but in the opposite direction.

STEERING ANGLE

Since the steering tie-rod influences the FBRC height it follows that steering input also plays a role. Every steering input moves the steering tie-rod and that may change the FBRC. Below is an example:

| Right | Side | Left | Side |
|----------|----------|---------------|-----------|
| Steering | Roll Cn | tr Steering | Roll Cntr |
| Angle | Height | Angle | Height |
| degrees | mm | degrees | mm |
| 0.000 | 41.027 | 0.000 | 41.027 |
| 1.000 | 41.220 | -1.010 | 40.824 |
| 2.000 | 41.407 | -2.039 | 40.608 |
| 3.000 | 41.589 | -3.088 | 40.378 |
| 4.000 | 41.765 | -4.159 | 40.132 |
| 5.000 | 41.937 | -5.251 | 39.868 |
| Table 5: | Steering | Angle effects | 5 |

INDEPENDENT SUSPENSION

One advantage of the FBRC is that the left and right suspensions are **independent**. Each side contributes a jacking force and overturning torque independent of the other side. The kinematic roll center depends on **both** sides in a complex way.

Looking at the left and right suspensions independently also makes it easier to consider asymmetric suspensions. Asymmetric suspensions are common in oval track racing. Many texts also texts acknowledge that a symmetric car becomes asymmetric as soon as the vehicle rolls [6].

Most authors refer to a symmetric car, which is common in the production car world. I suspect the roll center came into usage because it provided a convenient geometric interpretation of the distribution of lateral forces. But it is not applicable to asymmetric suspensions. The convenient interpretation has been given more importance than the underlying reality.

With a symmetric suspension both sides share a common Force Application Point. Lateral forces from the tires may be combined without knowing the distribution of forces between the tires. Thus the overturning moment can be calculated without knowing the distribution of tire forces. This is very convenient.

But the jacking forces have a different sign. Thus you must know the distribution of tire forces to determine the combined jacking force.

WHAT IS A ROLL CENTER

Dixon writes: "With detailed computer simulations that consider the forces in the individual suspension links it is not necessary to use the roll-centre concept. However. the roll-centre is a very useful idea, because the rollcentre height concisely summarises the effect of the links. With known roll-centre heights it is easy to calculate the roll angle and the load transfer at each of the front and rear axles. This is important in handling at large lateral accelerations because it affects the individual tyre vertical forces, and therefore their lateral forces. Hence the roll-centre height may be used as a summary of the load transfer characteristics of a suspension found by a detailed suspension analysis, or as the input specification for a simple handling simulation." [4]

Dixon continues "The S.A.E. defines the roll-centre in terms of forces, despite its kinematic name. A definition based on forces will be presented and used here. However, many authors introduce the roll-axis as an axis about which the vehicle actually rolls during cornering, the roll axis being the line joining the front and rear rollcentres. When a vehicle is actually moving on a road, the concept of a kinematic roll axis is difficult to justify in a precise way, especially for large lateral accelerations. Therefore the idea of the vehicle rolling about such an axis, although useful as a qualitative idea, should be treated rather cautiously, except in the special case of a stationary vehicle subject to loads in the laboratory." [4]

THE MYTH OF THE ROLL AXIS

The Roll Axis is part of every text but too many people believe the chassis actually rolls about the roll axis. There are several arguments to refute this:

- 1. The sprung mass does not simply *roll* in response to a lateral force; it *moves*. The jacking forces generated by the lateral force also produce a vertical force. This force will cause movement of the sprung mass rather than pure roll.
- 2. The springs, dampers and anti-roll bars determine the actual movement of the sprung mass. Very stiff springs on one side will cause less movement on that side. If you insist on determining a rotation point, a point which remains stationary, that rotation point will move toward the stiff spring.

THE PITCH CENTER

The same analysis can be applied in the longitudinal direction. Here the pitch center replaces the roll center.

It is interesting that anti-dive and anti-squat are often discussed as a percentage whereas the roll center is always a length. It would make more sense to discuss the roll center as an anti-roll percentage with 0% representing an instant center at ground level and 100% having the CP-IC line go through the center of gravity. The anti-roll percentage would scale better when discussing racing cars with a CG of 250 mm and large laden trucks with a CG height of more than a meter.

This illustrates the fact that the roll center is more useful for symmetric suspensions. The vehicle which has the same suspension front and rear is very rare.

| Tie-Ro | od FBR | C Force | Geometry | Force | Geo. |
|--------|---------|-----------|----------|--------|--------|
| height | t heigh | t est. | est. | 8 | est. |
| -40 | 100.18 | 3 100.183 | 99.998 | -0.960 | -0.970 |
| -30 | 103.78 | 0 103.780 | 103.629 | -0.971 | -0.982 |
| -20 | 107.46 | 3 107.463 | 107.348 | -0.982 | -0.994 |
| -10 | 111.23 | 4 111.234 | 111.159 | -0.994 | -1.006 |
| 0 | 115.09 | 5 115.096 | 115.066 | -1.006 | -1.018 |
| 10 | 119.05 | 2 119.052 | 119.071 | -1.018 | -1.031 |
| 20 | 123.01 | 5 123.016 | 123.179 | -1.031 | -1.044 |
| 30 | 127.26 | 2 127.262 | 127.394 | -1.044 | -1.058 |
| 40 | 131.52 | 4 131.524 | 131.720 | -1.057 | -1.072 |

Table 5: Anti factor with rear steer with flat tie-rod

Note the magnitude of the forces going through the steering tie-rod. The tie-rod is seeing a force equal to the entire braking force. Forces on the links are much higher as they resist a longitudinal force with a narrow base.

CONCLUSION

The method described above reconciles the difference between kinematic roll center and the force-based. But since we usually calculate a roll center to determine a roll couple, which is based on forces, we might as well use the force-based roll center. The value of the kinematic roll center remains the graphical interpretation. It is easy to visualize a kinematic roll center. It is difficult to visualize the inverse of a six-by-six matrix.

But this analysis provides a graphical interpretation of the force-based roll center. In 2-dimensions it matches the kinematic roll center. In 3-dimensions we must consider the steering tie-rod, but that can be done graphically also.

Use the force-based roll center for accuracy. Use the kinematic roll center for understanding and as a way of determining roll center stability. [6]

The geometry estimate might be improved, but there seems little reason to invest the time since the Force-based roll center is readily available.

The RollCen program [9] used to create Figures 4, 5, 6, 7, 8, 9 and 12 is available from the author. The WinGeo3 program [10] produced the calculations used in Tables 1-5.

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CONTACT

Wm. C. Mitchell Software, 125 East Plaza Drive Suite 117, Mooresville, NC 28115 USA. 800-844-7296 from USA and Canada 704-660-0330 voice 704-663-0085 fax www.MitchellSoftware.com

APPENDIX

Suspension coordinates for the test case:

| Point | Х | Y | Z |
|--------------------|---------|---------|---------|
| Lower Arm forward | -80.000 | 240.000 | 155.000 |
| Lower ball joint | 0.000 | 729.292 | 160.000 |
| Lower Arm rearward | 80.000 | 240.000 | 165.000 |
| Upper Arm forward | -80.000 | 325.000 | 440.000 |
| Upper ball joint | 40.000 | 715.142 | 480.000 |
| Upper Arm rearward | 80.000 | 325.000 | 440.000 |
| Tie-rod outboard | 100.000 | 680.000 | 160.000 |
| Tie-rod inboard | 100.000 | 240.000 | 160.000 |
| Half-track | | 838.202 | |
| Tire diameter | | | 622.301 |