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OBSERVATIONS ON THE METHOD OF DETERMINING THE VELOCITY OF AIRSHIPS,

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OBSERVATIONS ON THE METHOD OF DETERMINING THE VELOCITY OF AIRSHIPS.*

If V is the velocity of the airships, v the velocity of the wind, the resultant V^1 will be the effective velocity at which the machine covers the distance.

The angle $VV^1 = \gamma$ will be the angle of drift, and the angle $V^1v = \alpha$ the angle of the wind with the direction of route.

We then have:

$$V^1 = V \cos\gamma + v \cos\alpha$$

$$k = \frac{v}{V} = \frac{\sin\gamma}{\sin\alpha}$$

From these formulas we obtain:

$$\sin\gamma = k \sin\alpha$$
$$\cos\gamma = \sqrt{1 - k^2 \sin^2\alpha}$$
$$\cos\alpha = \sqrt{1 - \frac{\sin^2\alpha}{k^2}}$$

Suppose that the distance ds is covered in the time dt, we shall have:

$$dt = \frac{ds}{V^1} = \frac{ds}{V \cos\gamma + v \cos\alpha} = \frac{(V \cos\gamma - v \cos\alpha)\,ds}{V^2 \cos^2\gamma - v^2 \cos^2\alpha} =$$

$$= \frac{(V \cos\gamma - v \cos\alpha)\,ds}{V^2 (\cos^2\gamma - k^2 \cos^2\alpha)} = \frac{(V \cos\gamma - v \cos\alpha)\,ds}{V^2 (1 - k^2)} =$$

$$= \frac{\cos\gamma - k \cos\alpha}{V(1 - k^2)}\,ds$$

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and therefore, in the time t the distance covered, s, will be

$$t = \int \frac{\cos \gamma - k \cos \alpha}{V (1 - k^2)} ds$$

Supposing the wind to be constant in magnitude and direction, the foregoing formula becomes:

$$t = \frac{1}{V (1 - k^2)} \int (\cos \gamma - k \cos \alpha) ds =$$

$$= \frac{1}{V (1 - k^2)} \left\{ \int \cos \gamma \, ds - \int \cos \alpha \, ds \right\}$$

If we take the axis x parallel to the direction of the wind and project the line of flight AB along x in A'B' = x, we shall have:

$$x = \int \cos \alpha \, ds$$

therefore

$$t = \frac{1}{V (1 - k^2)} \left\{ \int \cos \gamma \, ds - kx \right\}$$

If the route covered is in a closed circuit, that is, A coincides with B and A' with B', we shall have x = 0 and the result will be:

$$t = \frac{1}{V (1 - k^2)} \int \cos \gamma \, ds = \frac{1}{V (1 - k^2)} \int \sqrt{1 - k^2 \cos^2 \alpha} \, ds$$

The average speed at which the machine covers the distance will be:

$$W = \frac{s}{t} \quad \frac{V (1 - k^2) s}{\theta}$$

assuming that

$$\theta = \int \cos \gamma \, ds = \int \sqrt{1 - k^2 \cos^2 \alpha} \; ds$$

It follows that the ratio between the mean velocity at which the distance is covered and the velocity proper of the machine will be:

$$\frac{W}{V} = \frac{(1 - k^2)s}{\theta} = \Omega$$

We will now show that this ratio is always less than 1 when the velocity of the wind is less than the velocity proper of the machine, that is, $k < 1$. In point of fact:

$$\Omega = \frac{(1 - k^2)s}{\theta} = \sqrt{1 - k^2} \; \frac{s\sqrt{1 - k^2}}{\int \sqrt{1 - k^2 \cos^2 \alpha} \; ds} =$$

$$= \sqrt{1 - k^2} \; \frac{\int \sqrt{1 - k^2} \; ds}{\int \sqrt{1 - k^2 \cos^2 \alpha} \; ds}$$

But:

$$\sqrt{1 - k^2} < \sqrt{1 - k^2 \cos^2 \alpha} < 1$$

therefore

$$\frac{\int \sqrt{1 - k^2} \; ds}{\int \sqrt{1 - k^2 \cos^2 \alpha} \; ds} < 1$$

and

$$\Omega = \sqrt{1 - k^2} \; \frac{\int \sqrt{1 - k^2} \; ds}{\int \sqrt{1 - k^2 \cos^2 \alpha} \; ds} < 1$$

$$\frac{W}{V} = \Omega < 1$$

IF, THEN, WE MEASURE THE MEAN VELOCITY ALONG ANY CLOSED CIRCUIT WHATEVER, WE SHALL ALWAYS OBTAIN A VELOCITY LESS THAN THE VELOCITY PROPER OF THE AIRSHIP ALTHOUGH THE WIND MAY SOMETIMES BE FAVORABLE, SOMETIMES OPPOSED OR SOMETIMES BE BLOWING SIDEWAYS TO THE DIRECTION OF MOVEMENT.

Let us examine some particular cases:

Suppose first that the circuit covered is a polygon $A\ B\ C\ D\ E = s_1\ s_2\ s_3\ s_4\ s_5$.

Denote the values of α along the sides by α_1, α_2, α_3, α_4, α_5, and the values of the Angle of Drift by γ_1, γ_2, γ_3, γ_4, γ_5. We shall have:

$$\frac{W}{V} = \frac{(1 - k^2)s}{\Sigma \cos \gamma_1 s_1} = \frac{(1 - k^2)s}{\Sigma s_1 \sqrt{1 - k^2 \cos^2 \alpha_1}}$$

Suppose the polygon to two superposed sides AB and BA, we shall have:

$$W = \frac{s}{t} < V$$

that is, assume $AB = L$ and denote by t' and t'' the times in which the distances AB and BA are covered.

$$W = \frac{2\ L}{t' + t''} = \frac{L}{\left(\frac{t' + t''}{2}\right)} < V$$

Thus, if we divide the distance L by the mean value of the times in which L is covered either in the direct sense or in the opposite direction, we shall always find a velocity less than the velocity proper of the airship unless the wind is zero.

THE ANGLES OF DRIFT FOR BOTH LINES OF FLIGHT, DIRECT AND IN THE CONTRARY DIRECTION, WILL BE EQUAL.

Denoting them by γ, we shall have:

$$\frac{L}{t'} = V \cos \gamma + v \cos \alpha \qquad \frac{L}{t''} = V \cos \gamma - v \cos \alpha$$

Therefore

$$\frac{1}{2}\left(\frac{L}{t'} + \frac{L}{t''}\right) = V \cos \gamma$$

which means that the mean velocity of flight along both lines, direct and inverse, is equal to the velocity proper of the airship multiplied by the cosine of the angle of drift.

IN MEASURING THE VELOCITY PROPER OF AN AIRSHIP WHEN A CROSS WIND IS BLOWING, WHICH IS USUALLY THE CASE, IT IS ADVISABLE TO MEASURE THE ANGLE OF DRIFT BOTH FOR THE DIRECT AND INVERSE LINE OF FLIGHT. THERE SHOULD NOT BE MUCH DIFFERENCE BETWEEN THESE TWO ANGLES. WE TAKE THE AVERAGE OF THESE ANGLES AND IT IS ADVISABLE TO INTRODUCE A CORRECTION DUE TO DRIFT BY DIVIDING THE MEAN OF THE TWO VELOCITIES OF THE TWO LINES OF FLIGHT BY THE COSINE OF THE ANGLE OF DRIFT.

For this correction to be of value, the distance covered must be practically rectilinear and the angle of drift must be constant, thus showing that the wind is constant.

We can also obtain the velocity and direction of the wind from the velocity of the two lines of flight, direct and inverse, and from the angle of drift.

In fact, from the equations:

$$\frac{L}{t'} = V \cos\gamma + v\cos\alpha \qquad \frac{L}{t''} = V\cos\gamma$$

it follows that

$$V = \frac{1}{2\cos\gamma}\left(\frac{L}{t'} + \frac{L}{t''}\right) \qquad v = \frac{1}{2\cos\alpha}\left(\frac{L}{t'} - \frac{L}{t''}\right)$$

therefore

$$k = \frac{v}{V} = \frac{\cos\gamma}{\cos\alpha}\,\frac{\frac{L}{t'} - \frac{L}{t''}}{\frac{L}{t'} + \frac{L}{t''}} = \frac{\cos\gamma}{\sqrt{1 - \frac{\sin^2\gamma}{k^2}}}\left[\frac{\frac{L}{t'} - \frac{L}{t''}}{\frac{L}{t'} + \frac{L}{t''}}\right.$$

from which we deduce

$$\sqrt{k^2 - \sin^2\gamma} = \cos\gamma\left[\frac{\frac{L}{t'} - \frac{L}{t''}}{\frac{L}{t'} + \frac{L}{t''}}\right]$$

$$k = \sqrt{\sin^2\gamma + \cos^2\gamma\left[\frac{\frac{L}{t'} - \frac{L}{t''}}{\frac{L}{t'} + \frac{L}{t''}}\right]^2}$$

and therefore

$$v = V\sqrt{\sin^2\gamma + \cos^2\gamma\left[\frac{\frac{L}{t'} - \frac{L}{t''}}{\frac{L}{t'} + \frac{L}{t''}}\right]^2} =$$

$$= \frac{1}{2\cos\gamma}\left[\frac{L}{t'} + \frac{L}{t''}\right]\sqrt{\sin^2\gamma + \cos^2\gamma\left[\frac{\frac{L}{t'} - \frac{L}{t''}}{\frac{L}{t'} + \frac{L}{t''}}\right]^2}$$

which means that

$$v = \frac{1}{2}\sqrt{\left[\frac{L}{t'} - \frac{L}{t''}\right]^2 + \left[\frac{L}{t'} + \frac{L}{t''}\right]^2 \tan^2\gamma}$$

We then have

$$\sin\alpha = \frac{\sin\gamma}{k} = \frac{\sin\gamma}{\sqrt{\sin^2\gamma + \cos^2\gamma\left[\frac{\frac{L}{t'} - \frac{L}{t''}}{\frac{L}{t'} + \frac{L}{t''}}\right]^2}} =$$

$$= \frac{\left[\frac{L}{t'} + \frac{L}{t''}\right]\tan\gamma}{\sqrt{\left[\frac{L}{t'} - \frac{L}{t''}\right]^2 + \left[\frac{L}{t'} + \frac{L}{t''}\right]^2\tan^2\gamma}}$$

Suppose that the distance covered forms a circle of radius R. In such a case we shall have:

$$\theta = \int\sqrt{1 - k^2\cos^2\alpha}\, ds = R\int_0^{2\pi}\sqrt{1 - k^2\cos^2\alpha}\, d\alpha =$$

$$= R\int_0^{2\pi}\frac{1 - k^2\cos^2\alpha}{\sqrt{1 - k^2\cos^2\alpha}}\, d\alpha =$$

$$= R \left\{ \int_0^{2\pi} \frac{d\ \alpha}{\sqrt{1 - k^2 \cos^2 \alpha}} - k^2 \int_0^{2\pi} \frac{\cos^2 \alpha\ d\ \alpha}{\sqrt{1 - k^2 \cos^2 \alpha}} \right\}$$

Now:

$$\int_0^{2\pi} \frac{d\ \alpha}{\sqrt{1 - k^2 \cos^2 \alpha}} = 4 \int_0^{2/\pi} \frac{d\ \alpha}{\sqrt{1 - k^2 \sin^2 \alpha}} = 4\ K$$

(elliptical integral of 1st grade)

$$\int_0^{2\pi} \frac{\cos^2\ \alpha\ d\ \alpha}{\sqrt{1 - k^2 \cos^2\ \alpha}} = 4 \int_0^{\pi/2} \frac{\sin^2\ \alpha\ d\ \alpha}{\sqrt{1 - k^2 \sin^2\ \alpha}} = 4\ E$$

(elliptical integral of 2nd grade)

therefore

$$\theta = 4\ R\ (K - k^2 E)$$

and

$$t = \frac{4\ R\ (K - k^2 E)}{V\ (1 - k^2)}$$

whence

$$V = \frac{4\ R\ (K - k^2 E)}{t\ (1 - k^2)}$$

Suppose that the airship flies successively along the two routes $M_1 N_1$, $M_2 N_2$ and that the velocity of the wind is such that it is represented by v. The axis of the airship is represented by V' when sailing over the first route and by V" when sailing over the second route.

The two vectors v will be equivalent to each other, while the two vectors V' and V" denoting the velocity proper of the airship will have different directions, but will have the same modulus V if the absolute velocity of the airship V is maintained at the same magnitude during the flight over the two routes.

We will take the geographical direction S N (South-North) and will represent the angles as follows:

$(M_1 N_1) . (SN) = \theta_1$ $\quad$ $(M_2 N_2) . (SN) = \theta_2$

$V^1 . (SN) = X_1$ $\quad$ $V^2 . (SN) = X_2$

$v_1 (SN) = \rho$ $\quad$ $v . (SN) = \rho$

Lastly, we denote by V_1 and V_2 the effective speed at which the airship covers the two routes; by γ_1 and γ_2 the two angles of drift; by α_1 and α_2 the angles formed by the wind with each of the two routes, and by θ the angle formed by the two routes.

We have:

$$\gamma_1 = \theta_1 - X_1 \qquad (1)$$

$$\gamma_2 = \theta_2 - X_2 \qquad (2)$$

$$\alpha_1 = \rho - \theta_1 \qquad (a)$$

$$\alpha_2 = \rho - \theta_2 \qquad (a')$$

$$\theta = \theta_2 - \theta_1 = \alpha_1 - \alpha_2 \qquad (3)$$

$$\frac{v}{V} = \frac{\sin \gamma_1}{\sin \alpha_1} = \frac{\sin \gamma_2}{\sin \alpha_2} \qquad (4)$$

$$\left.\begin{aligned} V_1 &= V \cos \gamma_1 + v \cos \alpha_1 \\ V_2 &= V \cos \gamma_2 + v \cos \alpha_2 \end{aligned}\right\} \quad (5)$$

From (4) we obtain:

$$v = V \frac{\sin \gamma_1}{\sin \alpha_1} = V \frac{\sin \gamma_2}{\sin \alpha_2} \qquad \text{(b)}$$

therefore (5) becomes:

$$\left.\begin{aligned} V_1 &= V(\cos \gamma_1 + \sin \gamma_1 \ \text{cotan} \ \alpha_1) \\ V_2 &= V(\cos \gamma_2 + \sin \gamma_2 \ \text{cotan} \ \alpha_2) \end{aligned}\right\} \qquad (5')$$

But from (4) we have also:

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{\sin \gamma_1}{\sin \gamma_2} \qquad\qquad \frac{\sin \alpha_2}{\sin \alpha_1} = \frac{\sin \gamma_2}{\sin \gamma_1}$$

and from (3)

$$\alpha_1 = \theta + \alpha_2 \qquad\qquad \alpha_2 = \alpha_1 - \theta$$

therefore

$$\frac{\sin \gamma_1}{\sin \gamma_2} = \frac{\sin(\theta + \alpha_2)}{\sin \alpha_2} = \sin \theta \ \text{cotan} \ \alpha_2 + \text{cos}$$

$$\frac{\sin \gamma_2}{\sin \gamma_1} = \frac{\sin(\alpha_1 - \theta)}{\sin \alpha_1} = \cos \theta - \sin \theta \ \text{cotan}$$

from which it follows that:

$$\text{cotan} \ \alpha_2 = \left[\frac{\sin \gamma_1}{\sin \gamma_2} - \cos \theta\right] \frac{1}{\sin \theta} \qquad \text{(c)}$$

$$\text{cotan} \ \alpha_1 = \left[\cos \theta - \frac{\sin \gamma_2}{\sin \gamma_1}\right] \frac{1}{\sin \theta} \qquad \text{(c')}$$

and substituting in (5')

$$V_1 = V\left[\cos\gamma_1 + \sin\gamma_2\left(\cos\theta - \frac{\sin\gamma_2}{\sin\gamma_1}\right)\frac{1}{\sin\theta}\right] =$$

$$= \frac{V(\cos\gamma_1 \sin\theta + \sin\gamma_1 \cos\theta - \sin\gamma_2)}{\sin\theta} =$$

$$= \frac{V(\sin[\theta + \gamma_1] - \sin\gamma_2)}{\sin\theta} =$$

$$= \frac{2V\sin\left[\dfrac{\theta + \gamma_1 - \gamma_2}{2}\right]\cos\left[\dfrac{\theta + \gamma_1 + \gamma_2}{2}\right]}{\sin\theta} \tag{6}$$

$$V_2 = V\left[\cos\gamma_2 + \sin\gamma_2\left(\frac{\sin\gamma_1}{\sin\gamma_2} - \cos\theta\right)\frac{1}{\sin\theta}\right] =$$

$$= \frac{V(\cos\gamma_2 \sin\theta - \cos\theta \sin\gamma_2 + \sin\gamma_1)}{\sin\theta} =$$

$$= \frac{V(\sin[\theta - \gamma_2] + \sin\gamma_1)}{\sin\theta} =$$

$$= \frac{2V\sin\left[\dfrac{\theta - \gamma_2 + \gamma_1}{2}\right]\cos\left[\dfrac{\theta - \gamma_2 - \gamma_1}{2}\right]}{\sin\theta} \tag{7}$$

From formulas (6) and (7) we obtain:

$$V = \frac{V_1 \sin \theta}{2 \sin \dfrac{\theta + \gamma_1 - \gamma_2}{2} \cos \dfrac{\theta + \gamma_1 + \gamma_2}{2}} =$$

$$= \frac{V_2 \sin \theta}{2 \sin \dfrac{\theta - \gamma_2 + \gamma_1}{2} \cos \dfrac{\theta - \gamma_2 - \gamma_1}{2}} \qquad (8)$$

Now, from (1), (2), and (3) it follows that:

$$\theta + \gamma_1 - \gamma_2 = \theta_2 - \theta_1 + \theta_1 - X_1 - \theta_2 + X_2 = X_2 - X_1$$

$$\theta + \gamma_1 + \gamma_2 = \theta_2 - \theta_1 + \theta_1 - X_1 + \theta_2 - X_2 =$$

$$= 2\,\theta_2 - X_1 - X_2$$

$$\theta - \gamma_1 - \gamma_2 = \theta_2 - \theta_1 - \theta_1 + X_1 - \theta_2 + X_2 =$$

$$= X_1 + X_2 - 2\,\theta_1$$

therefore (8) becomes:

$$V = \frac{V_1 \sin (\theta_2 - \theta_1)}{2 \sin \left[\dfrac{X_2 - X_1}{2}\right] \cos \left[\theta_2 - \dfrac{X_1 \; X_2}{2}\right]} =$$

$$= \frac{V_2 \sin(\theta_2 - \theta_1)}{2 \sin\left[\frac{X_2 - X_1}{2}\right] \cos\left[\theta_1 - \frac{X_1 + X_2}{2}\right]} \qquad (A)$$

Thus, wishing to obtain the absolute velocity of the airship by knowing the speed at which the two routes are covered, we have only to determine the geographical direction of the routes which we locate from the map, and the angles of the routes as given by the compass, after correcting for the variation (the algebraical sum of the local magnetic declination and the deviation).

The determination of the constancy of the wind along the two routes is given by equalizing the two values of V given by the second and third terms of (A).

We can also obtain the direction and magnitude of the wind, since from formulas (c) and (c') we can have the angles α_1 and α_2 and therefore from (a) and (a') we shall have ρ in two ways.

Then by applying formula (b) we obtain the velocity v of the wind in two ways also.

These various ways of determining the values of v and ρ may also serve for determining the constancy of the wind on the two routes.

PRACTICAL EXAMPLE.

DETERMINATION OF THE VELOCITY OF THE AIRSHIP M.6.

We give here two examples of the determination of the speed of the airship M.6 made on board along a straight line of known length, traversed in both directions, along the railway line Signa-Florence between the two localities "Sardigna" and "Castel-Paoli" (length 6.925 km.)

The measurements made and the results obtained are summarized in the four following tables which contain:

The first, all the elements concerning the measurements and the resulting values of the velocity observed in the first determination.

The second, all the data required for computing the angle of drift presented by the route of the airship with respect to the orientation of the track while the measurements were being taken.

The third and fourth tables give similar data concerning the second determination.

These latter data serve for passing from the observed value of the velocity to the true value, the effect of the wind being eliminated in each case. The data referring to the computation of the angle of drift were deduced from the compass readings while the measurements were being taken, corrections being made for deviations due to local magnetic conditions.

From the results obtained in the two ascensions, we may conclude that the airship M.6 had a mean velocity of 69.2 with the regime of the engines maintained constant during the experiments here recorded.

FIRST DETERMINATION

Altitude	Pressure	Temperature	Route	Dist.	Angle of route by compass	Time	Speed per hour	Mean speed
m.	m/m			km.			km.	km.
210	740	27.2°	C. Paoli-Sardigna	6.925	100°	4m46.9s	86.8	68.60
220	741	27.5°	Sardigna-C. Paoli	"	275°	8m16.4s	50.3	

ANGLE OF DRIFT

Route followed	Angle of route by compass	Actual angle of route.	Orientation of route	Angle of drift	Mean angle of drift.
C. Paoli-Sardigna	100°	84°	79°	5°	7° 30'
Sardigna-C. Paoli	275°	269°	259°	10°	

$$v = \frac{68.60}{\cos 7^\circ 30'} = 69.19 \text{ km.}$$

SECOND DETERMINATION

Altitude	Pressure	Temperature	Route	Dist.	Angle of route by compass	Time	Speed per hour	Mean Speed.
m	m/m			km.			km.	km.
320	-	15.3°	C.Paoli-Sardigna	6.925	95°	6m3.3s	68.68	69.15
285	-	15.2°	Sardigna-C. Paoli	"	265°	5m58.1s	69.62	

ANGLE OF DRIFT.

Route followed	Angle of route by compass	Actual angle of route	Orientation of route	Angle of drift	Mean angle of drift
C. Paoli-Sardigna	95°	80°	79°	1°	2° 1/2
Sardigna-C. Paoli	265°	263°	259°	4°	

$$v = \frac{69.16}{\cos 2^{\circ}\,30'} = 69.22$$